Preparatory Test for Entrance Examination in Mathematics (PTEEM) 2022

Organised by

C.P.S.M. **Class -XI**

Time: 2 hrs. **Subject: Mathematics** **Full Marks: 100**

INSTRUCTION: (1) Write your Name, Class, Name of School and Roll No. in the appropriate places of the answer-sheet. (2) Find out which of the answers appears to you to be correct or the best. There are four reactangles on the answer-sheet corresponding to each question no. (a), (b), (c) & (d). Now mark the rectangle below the letter of the selected answer in the answer-sheet by blackening distinctly with a H.B. pencil as shown here ☐☐■☐, if (c) is the correct answer, (3) Don't write anything on the question paper. (4) Don't underline or tick the answer on the question paper. Submit the answer-sheet only after the examination. (5) You may use additional blank sheet for any rough work, if necessary. (6) Dont waste time for answerwing a question which appears difficult to you, better try next question.

Category-I (Q.1 to Q. 50)

Only one answer is correct. Correct answer will fetch full mark 1. For incorrect answer or any combination of more than one answer, 1/4 marks will be deducted.

1. If Set $A = \{1, 3, 5, 7, 9\}$ and Set $B = \{2, 3, 5, 7, 11\}$, then $A \triangle B$ is equal to

   (a) $\{3, 5, 7\}$ (b) $\{1, 2\}$ (c) $\{9, 11\}$ (d) $\{1, 2, 9, 11\}$

2. The range of the function $f(x) = \dfrac{\sin\left(\pi\left[x^2+1\right]\right)}{x^4+1}$ where [ ] is greatest integer function, is

   (a) $[0, 1]$ (b) $[-1, 1]$ (c) $\{0\}$ (d) none of these

3. If $\sqrt{3}$, A, and $\sqrt{2}$ are in AP then $\dfrac{\sqrt{3}+\sqrt{2}}{2}$ is greater than or equal to

   (a) $\sqrt{5}$ (b) $\sqrt{6}$ (c) $\sqrt{8}$ (d) none of these

4. If $\displaystyle\sum_{k=1}^{n}\left(\sum_{m=1}^{k} m^2\right) = an^4 + bn^3 + cn^2 + dn + e$ then,

   (a) $a = \dfrac{1}{12}$ (b) $b = \dfrac{1}{2}$ (c) $d = \dfrac{1}{5}$ (d) $e = 1$

5. If $|z+1| = \sqrt{2}\,|z-1|$ then the locus described by the point z in the argund diagram is a

(a) straight line (b) circle (c) parabola (d) none of these

6. Let $f:(4, 6) \to (6, 8)$ be a function defined by $f(x) = x + \left[\frac{x}{2}\right]$, where [ ] denotes the greatest integer function, then $f^{-1}(x)$ is equal to

(a) $x - \left[\frac{x}{2}\right]$ (b) $-x-2$ (c) $x-2$ (d) $\dfrac{1}{x+\left[\frac{\pi}{2}\right]}$

7. The complex numbers z_1 and z_2 are such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\left(\dfrac{z_1+z_2}{z_1-z_2}\right)$ may be

(a) zero (b) real and positive

(c) real negative (d) purely imaginary

8. The value of $\left(\dfrac{1+i}{\sqrt{2}}\right)^8 + \left(\dfrac{1-i}{\sqrt{2}}\right)^8$ is equal to

(a) 4 (b) 6 (c) 8 (d) 2

9. The values of 'a' for which $x^2 + ax + \sin^{-1}(x^2-4x+5) + \cos^{-1}(x^2-4x+5) = 0$ has atleast one solution is

(a) -2 (b) $-2+\pi$ (c) $-\dfrac{\pi}{4}$ (d) $-2-\dfrac{\pi}{4}$

10. If the absolute value of the difference of roots of the equation $x^2 + px + 1 = 0$ exceeds $\sqrt{3p}$ then

(a) $p < -1$ or $p > 4$ (b) $p > 4$ (c) $-1 < p < 4$ (d) $0 \leq p < 4$

11. The number of ways in which the sum of upper faces of four distinct dices can be six is

(a) 10 (b) 4 (c) 6 (d) 7

12. f : {1, 2, 3, 4, 5} $\rightarrow$ {x, y, z, t} then the total number of onto functions is equal to

(a) 242 (b) 245 (c) 102 (d) 240

13. The ratio of the co-efficient of x^{15} to the term independent of x in $\left(x^2 + \frac{2}{x}\right)^{15}$ is

(a) 12 : 32 (b) 1 : 32 (c) 32 : 12 (d) 32 : 1

14. Let n be a positive integer such hat $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$ then $\sum_{r=0}^{2n} a_r$ is

(a) 3^n (b) 3^{n-1} (c) $\frac{3^n}{2}$ (d) none of these

15. If $= \frac{1+\sin 2x}{1-\sin 2x} = \cos^2(a+x),\ x \in R \sim \left(x\pi + \frac{\pi}{4}\right)$, $x \in N$, then a is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) none of these

16. If $s = \cos^2\frac{\pi}{n} + \cos^2\frac{2\pi}{n} + \ \ldots\ + \frac{\cos^2(n-1)\pi}{n}$, then s equals

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n-1}{2}$ (c) $\frac{n-2}{2}$ (d) $\frac{n}{2}$

17. The set of values of x for which $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$ is

(a) ϕ (b) $\left\{\frac{\pi}{4}\right\}$

(c) $\left\{n\pi + \frac{\pi}{4}, n = 1, 2, 3, \ldots\right\}$ (d) $\left\{2n\pi + \frac{\pi}{4}, n = 1, 2, 3, \ldots\right\}$

18. If $a^2 = b^3 = c^5 = d^6$, then value of $\log_d(abc)$ is

(a) $\frac{31}{6}$ (b) $\frac{31}{3}$ (c) $\frac{31}{5}$ (d) $\frac{31}{2}$

19. What will be the remainder if 2^{2003} is divided by 17

(a) 8 (b) 9 (c) 10 (d) 11

20. Value of $3^{\frac{1}{2}} \cdot 9^{\frac{1}{4}} \cdot 27^{\frac{1}{8}}, \ \ldots \ \infty$ is

(a) 7 (b) 9 (c) 18 (d) 27

21. Value of sin 10° + sin 50° – sin 70° is

(a) –1 (b) +1 (c) $\frac{1}{2}$ (d) 0

22. The value of $\lim_{x \to 0} \frac{1}{x} \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is

(a) 2 (b) ∞ (c) does not exist (d) none of these

23. $\lim_{x \to 1} \frac{\int_1^x |t-1| \, dt}{\sin(x-1)}$ is equal to

(a) 0 (b) 1 (c) – 1 (d) none of these

24. For a symmetrical distribution $Q_1 = 25$ and $Q_3 = 45$, then median is

(a) 28 (b) 35 (c) 30 (d) 40

25. The mean deviation from the mean of the AP $a, a + d, a + 2d, \ldots a + 2nd$ is

(a) $n(n + 1)d$ (b) $\frac{n(n+1)d}{2n+1}$ (c) $\frac{n(n+1)d}{2n}$ (d) $\frac{n(n-1)d}{(2n+1)}$

26 . If $y = \log\sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is given by

(a) ∞ (b) 1 (c) 0 (d) $\frac{1}{2}$

27. The area of quadrilateral whose vertices are (1, 1), (3, 4), (5, –2) and (4, –7) is

(a) 41 sq units (b) $\frac{41}{2}$ sq units (c) $\frac{31}{2}$ sq units (d) 7 sq units

28. If in a triangle ΔABC, $\angle B = 90°$ then $\tan^2\left(\frac{A}{2}\right)$ is equal to

(a) $\frac{b-c}{b+c}$ (b) $\frac{b+c}{b-c}$ (c) $\frac{b-2c}{b+c}$ (d) none of these

29. If the chord of contact of tangents drawn from the point (h, k) to the circle $x^2 + y^2 = a^2$ subtends a right angle at the centre then $h^2 + k^2$ is equal to

(a) $\frac{a^2}{2}$ (b) a^2 (c) $2a^2$ (d) none of these

30. If the circles $x^2 + y^2 + 2x + 2ay + 6 = 0$ and $x^2 + y^2 + 4ay + a = 0$ intersects orthogonally, then vlaue of a is equal to

(a) $\frac{1 \pm 4\sqrt{6}}{8}$ (b) $\frac{1 \pm 4\sqrt{7}}{8}$ (c) $\frac{1 \pm \sqrt{97}}{8}$ (d) $\frac{1 \pm 2\sqrt{6}}{8}$

31. If (2, –8) is at an and of a focal chord of the parabola $y^2 = 32x$, then the co-ordinate of the other and of the chord is

(a) (8, –2) (b) (16, 32) (c) (32, 32) (d) none of these

32. If the normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ cuts the parabola again at $(aT^2, 2aT)$ then

(a) $-2 \leq T \leq 2$ (b) $T \in (-\infty, -8) \cup (8, \infty)$

(c) $T^2 < 8$ (d) $T^2 \geq 8$

33. The locus of the points of trisection of the double ordinates of parabola $y^2 = 4ax$ is

(a) $y^2 = ax$ (b) $9y^2 = 4ax$

(c) $9y^2 = ax$ (d) $y^2 = 9ax$

34. The eccentricity of an ellipse whose pair of conjugate diameter are $y = x$ and $3y = -2x$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) none of these

35. Equation of a rectangular hyperbola whose asymptotes are $x = 3$ and $y = 5$ and passing through (7, 8), is

(a) $xy - 3y + 5x + 3 = 0$ (b) $xy - 3y + 4x + 3 = 0$

(c) $xy - 3y + 5x - 3 = 0$ (d) $xy - 3y - 5x + 3 = 0$

36. $\frac{3}{4}+\frac{15}{16}+\frac{63}{64}+ \dots +n$ terms is equal to

(a) $n - \frac{1}{3}4^{n} - \frac{1}{3}$ (b) $n + \frac{1}{3}4^{-n} - \frac{1}{3}$ (c) $n + \frac{1}{3}4^{n} - \frac{1}{3}$ (d) $n - \frac{1}{3}4^{-n} + \frac{1}{3}$

37. Find the image of the point (3, 8) with respect of the line $x + 3y = 7$ assuming the line to be plane mirror.

(a) (1, 4) (b) (–1, –4) (c) (–1, 4) (d) (1, – 4)

38. If the points (– 2, 0), $\left(-1, \frac{1}{\sqrt{3}}\right)$ and $(\cos\theta, \sin\theta)$ are collinear, then the number of values of $\theta \in [0, 2\pi]$ is

(a) 0 (b) 1 (c) 2 (d) infinite

39. The derivative of $\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$ is

(a) $\frac{2}{(1+x)^2}$ (b) $\frac{-2}{(1-x)^2}$ (c) $\frac{-1}{(1-x)^2}$ (d) $\frac{3}{(1-x)^2}$

40. $\sin\frac{2\pi}{7}+\sin\frac{4\pi}{7}+\sin\frac{8\pi}{7}$ is equal to

(a) $\frac{\sqrt{7}}{2}$ (b) $\sqrt{7}$ (c) 2 (d) $\frac{\sqrt{7}}{4}$

41. If $A = \{y : y = 2x, x \in N\}$, $B = \{y : y = 2x - 1, x \in N\}$ then $(A \cap B)'$ is

(a) A (b) B (c) ϕ (d) $\cup$

42. The sum of the n terms of the series $\frac{4}{1\cdot2\cdot3}+\frac{5}{2\cdot3\cdot4}+\frac{6}{3\cdot4\cdot5}+\ldots$ is

(a) $\frac{n+3}{n(n+1)(n+2)}$ (b) $\frac{n(n+1)}{n(n+1)(n+2)}$ (c) $\frac{5}{4}-\frac{2n+5}{2(n+1)(n+2)}$ (d) None of these

43. If $z=\frac{\cos\theta+i\sin\theta}{\cos\theta-i\sin\theta}$, $\frac{\pi}{4}<\theta<\frac{\pi}{2}$ then arg(z) is

(a) 2θ (b) $2\theta-\pi$ (c) $\pi+2\theta$ (d) None of these

44. *A* and *B* throw a dice each. The probability that *A*'s throw is not greater than *B*'s throw is

(a) $\frac{7}{12}$ (b) $\frac{5}{12}$ (c) $\frac{1}{6}$ (d) $\frac{1}{2}$

45. The probability that $\sin^{-1}(\sin x)+\cos^{-1}(\cos y)$ is an integer, $x, y \in \{1, 2, 3, 4\}$, is

(a) $\frac{1}{16}$ (b) $\frac{3}{16}$ (c) $\frac{15}{16}$ (d) None of these

46. $\lim_{x\to0}\left\{\frac{\log_e(1+x)}{x^2}+\frac{x-1}{x}\right\}$ is equal to

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) None of these

47. If $f(x)=1-\frac{1}{x}$ then $f\left\{f\left(\frac{1}{x}\right)\right\}$ is

(a) $\frac{1}{x}$ (b) $\frac{1}{1+x}$ (c) $\frac{x}{x-1}$ (d) $\frac{1}{x-1}$

48. The value of $(0.2)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\right)}$ is

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 4

49. If $z = x + iy$ and $|z - 2 + i| = |z - 3 - i|$ then locus of z is

(a) $2x + 4y - 5 = 0$ (b) $2x - 4y - 5 = 0$

(c) $x + 2y = 0$ (d) $x - 2y + 5 = 0$

50. If α amd β are the roots of equations $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + q = 0$ then the roots of $x^2 - 4qx + 2q^2 - r = 0$ are always

(a) both non-real (b) both positive (c) both real (d) opposite in sign

Category-II (Q.51 to Q. 65)

Only one answer is correct. Correct answer will fetch full mark 2. For incorrect answer or any combination of more than one answer, 1/2 marks will be deducted.

51. The value of $\lim_{n\to\infty}\left[\sqrt[3]{n^2 - n^3} + n\right]$ is

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

52. From any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, tangents are drawn to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2$. Then, area cut-off by the chord of contact on the asymptotes is equal to

(a) $\frac{a}{2}$ sq unit (b) ab sq unit (c) $2ab$ sq unit (d) $4ab$ sq unit

53. If the co-ordinates of the vertices of a ΔABC are $A(-1, 3, 2)$, $B(2, 3, 5)$ and $C(3, 5, -2)$, then $\angle A$ is equal to

(a) $45°$ (b) $60°$ (c) $90°$ (d) $30°$

54. If the latusrectum of a hyperbola through one focus subtends $60°$ angle at the other focus, then its eccentricity e is

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) $\sqrt{6}$

55. The value of the expression $\frac{\sin^3 x}{1+\cos x}+\frac{\cos^3 x}{1-\sin x}$ is

(a) $\sqrt{2}\cos\left[\frac{\pi}{4}-x\right]$ (b) $\sqrt{2}\cos\left[\frac{\pi}{4}+x\right]$ (c) $\sqrt{2}\sin\left[\frac{\pi}{4}-x\right]$ (d) None of these

56. Number of positive integer n less than 17, for which $n! + (b + 1)! + (n + 2)!$ is an integral multiple of 49, is

(a) 0 (b) 3 (c) 5 (d) 2

57. If α and β are the roots of the equation $8x^2 - 3x + 27 = 0$, then the value of $\left(\frac{\alpha^2}{\beta}\right)^{1/3}+\left(\frac{\beta^2}{\alpha}\right)^{1/3}$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{7}{2}$ (d) 4

58. If $x = \omega - \omega^2 - 2$, then the value of $x^2 + 3x^3 + 2x^2 - 11x - 6$ is

(a) 1 (b) –1 (c) 2 (d) None of these

59. If 1, $\log_y x$, $\log_z y$ and $-15\log_x z$ are in AP, then

(a) $z^3 = x$ (b) $x = y^2$ (c) $z^{-2} = y$ (d) None of these

60. The region of argand diagram defined by $|z - 1| + |z + 1| \leq 4$ is

(a) interior of an ellipse (b) exterior of a circle

(c) interior and boundary of an ellipse (d) None of these

61. $9^7 + 7^9$ is divisible by

(a) 6 (b) 24 (c) 64 (d) 72

62. If the equation $\tan\theta + \sec\theta = \sqrt{3}$, then the value of $\theta \in (0, 2\pi)$ is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{2}$ (c) $\frac{\pi}{6}$ (d) None of these

63. If the sides of a triangle are in ratio 3 : 7 : 8, then $R : r$ is equal to

(a) 2 : 7 (b) 7 : 2 (c) 3 : 7 (d) 7 : 3

64. If α, β and γ are the roots of the equation $x^3 - 3px^2 + 3qx - 1 = 0$, then the centroid of the triangle, whose vertices are $A\left(\alpha, \frac{1}{\alpha}\right), B\left(\beta, \frac{1}{\beta}\right)$ and $C\left(\gamma, \frac{1}{\gamma}\right)$ is

(a) $(p, -q)$ (b) (p, q)

(c) $(-p, q)$ (d) $(-p, -q)$

65. The area of the triangle formed by the lines $4x^2 - 9xy - 9y^2 = 0$ and $x = 2$ is equal to—

(a) $\frac{20}{3}$ sq units (b) 3 sq units

(c) $\frac{10}{3}$ sq units (d) 2 sq units

Category-III (Q.66 to Q. 75)

One or more answer(s) is (are) correct. Correct answer (s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked/actual number of correct answers.

66. The equation of the tangent of the parabola $y^2 = 9x$ which goes through the point (4, 10), is

(a) $x + 4y + 1 = 0$ (b) $9x + 4y + 4 = 0$

(c) $x - 4y + 36 = 0$ (d) $9x - 4y + 4 = 0$

67. Extremities of the latusrectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ having a given major axis $2a$, lies on

(a) $x^2 = a(a - y)$ (b) $x^2 = a(a + y)$

(c) $y^2 = a(a + x)$ (d) $y^2 = a(a - x)$

68. The equation $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ represents a hyperbola

(a) the length of whose transverse axis is $4\sqrt{3}$

(b) the length of whose conjugate axis is 4

(c) whose centre is (1, 2)

(d) whose eccentricity is $\sqrt{\frac{19}{3}}$

69. If $f(x) = e^{[\cot x]}$, where [y] represents the greatest integer less than or equal to y, then

(a) $\lim_{x \to \pi/2^+} f(x) = 1$

(b) $\lim_{x \to \pi/2^+} f(x) = \frac{1}{e}$

(c) $\lim_{x \to \pi/2^-} f(x) = \frac{1}{e}$

(d) $\lim_{x \to \pi/2^-} f(x) = 1$

70. If A and B are two independent events such that $P(\overline{A} \cap B) = \frac{2}{5}$ and $P(A \cap \overline{B}) = \frac{1}{6}$, then $P(B)$ is

(a) $\frac{4}{5}$ (b) $\frac{1}{6}$ (c) $\frac{1}{5}$ (d) $\frac{5}{6}$

71. If sum of the first three consecutive terms of an AP is 9 and the sum of their squars is 35. Then, sum to n terms of the series is

(a) $n(n + 1)$ (b) n^2 (c) $n(4 - n)$ (d) $n(6 - n)$

72. One vertex of the triangle of maximum area that can be inscribed in the curve $|z - 2i| = 2$, is $2 + zi$, remaining vertices is/are

(a) $-1 + i(2 + \sqrt{3})$

(b) $-1 - i(2 + \sqrt{3})$

(c) $-1 + i(2 - \sqrt{3})$

s(d) $-1 - i(2 - \sqrt{3})$

73. Let f, $g : R \rightarrow R$ be defined, respectively by, $f(x) = x + 1$ and $g(x) = 2x - 3$ then,

(a) $(f + g)(x) = 3x - 2$

(b) $(f - g)(2) = 6$

(c) $\left(\frac{f}{g}\right)(2) = 3$

(d) $(f - g)(x) = 4 - x$

74. The sum of the series $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{2}{25}\right) + \tan^{-1}\left(\frac{1}{18}\right) + \ldots$ is

(a) $\tan^{-1}(3)$

(b) $\cot^{-1}\left(\frac{1}{3}\right)$

(c) $\tan^{-1}\left(\frac{1}{3}\right)$

(d) None of these

75. The roots of the equation $\left(a+\sqrt{b}\right)^{x^2-15} + \left(a-\sqrt{b}\right)^{x^2-15} = 2a$ where $a^2 - b = 1$ are

(a) ± 3

(b) ± 4

(c) $\pm \sqrt{14}$

(d) $\pm \sqrt{5}$