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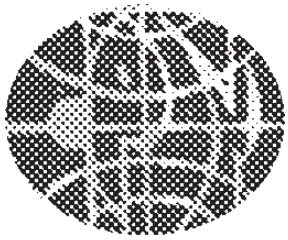

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CPSM: Challenges in Mathematics Education
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Contents

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Retrieving Sir Asutosh's Atributes From His Diary

Satyabachi Sar*

[This article is specially written and dedicated in memory of Dr. S. N. Giri, the principal architect of CPSM]

Attributes and qualities of Sir Asutosh Mukhopadhyay can be viewed from different angles considering his accomplishments and achievement as an able administrator, unique educationist, exceptional visionary, eminent judge, grand orator, mathematical genius, tireless propagator of mathematical research, distinguished bibliophile and above all a wonderful human being. His qualities like humility, greatness, patriotism, respect for cultural heritage, and love for literature rouse in every one an amazing feeling with great admiration and reverence.

Whichever field had got Asutosh's touch glittered brightly and drawn attention and admiration from all. No doubt, he was a versatile genius and. Asutosh himself was aware of the fact. But he always believed in perseverance, hard work, dedication and modesty which he showed through his activity.

The following portions taken from the entries of Asutosh's diary would reveal his patience, commitment, diligence and humility.

May 31, 1883: Procured from Cathedral Library a set of Brewster's Life of Newton and made up my mind to take Newton as a subject of my discourse.

June 1, 1883: Began to write out my intended speech on "Newton" ; worked on it whole afternoon, but made comparatively little progress; the hardest part of the speech is the opening passage which is intended to rivet the attention of the audience, of course, the peroration is still harder.

June 2, 1883: Wrote a few lines about Newton, could not work hard;

June 3, 1883: Read Brewster's Life of Newton, thus thoroughly prepared myself for the intended speech; finished it up at a single sitting, it is hardly possible to rest back the pen when the feelings get warmed; perhaps some passages are too abstrusely mathematical to be fully understood by my audience; however I have simplified my remarks as much as lay in my power.

June 4, 1883: Began to get by heart the speech on Newton; the style is now manageable and I can now hope for actually delivering it.

June 5, 1883: Rose at 4.50 a.m. Got by heart the speech on Newton.

June 6, 1883: Rose at 5.5 a.m. Got by heart the speech on Newton .

June 11, 1883: got by heart the on Newton, the portions that I had learned before are already dim; I have lost my former fluency.

June 12, 1883: Rose at 5.30 a.m. At noon, got by heart the speech on Newton and finished it together; delivered twice; if I can do so before the public, it will not be a very bad thing.

June 13, 1883: Rose at 5 a.m. Got by heart the speech on Newton.

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June 14, 1883: Read the speech on Newton; delivered at night before father, he said, it was a great improvement upon those of previous occasions; and he has no doubt that it will prove to be a decided success;

June 18, 1883: Rose at 5a.m. Delivered the lecture on Newton.

June 19, 1883: Rose at 5a.m. Delivered the speech on Newton.

June20, 1883: Rose at 4.50 a.m. Delivered before father the address on Newton; which I shall have to deliver in the ordinary meeting of our Club on Saturday next.

June 21, 1883: Rose at 5 a.m. Delivered before father the address on Newton; I am gradually gaining fluency but not hurry;

June 22, 1883: Rose at 5 a-m. Delivered before father the lecture on Newton as slowly as I could. The first thing to be avoided is hurry and the indistinctness due thereto.

June 23, 1883: Rose at 5a.m. Delivered before father the speech on Newton……. The meeting of our Club took place at 2 p.m.; Prof. Percival presided. Students from all the classes were present; I delivered my speech on Newton; it was a quite success; but I do I really deserved the unqualified praise showered on me by the President! I doubt whether I shall ever deserve that much.

Only these entries show that how much importance Asutosh attached to any of his duties, how much hard work he did for perfection in his performance, how much sincere in his preparation of a work and also reveal how humble he was.. He did never indulge any sort of boasting and negligence.

Asutosh was a true patriot in every respect. He believed in national dress and lived a life with utmost simplicity. Dressed in a most ordinary dhoti and a short coat he traveled different parts of India as a member of the Sadler Commission. In Calcutta High Court he used to change into dhoti after finishing his day's work as a judge. Though he was a true nationalist and had great respect for Indian tradition, he had no false vanity. The fact is evident from his address at Mysore. He said, 'we cannot sit on the lovely snow-capped pick of the Himalayas absorbed in contemplation of our glorious past…… We cannot waste precious time and strength in defense of theories and systems , which however valuable in their own days, have been swept away by the irresistible avalanche of world-wide changes. ….. Really, he combined, in rare perfection, reverence for Indian intellectual tradition with a passionate pioneering zeal for modern scholarship. He never traveled abroad and he never felt for himself the rhythm of the western world. 'He was anxious to bring India to the mid-current of modern intellectual life. We may narrate an interesting episode here.

After the Convocation Address in Mysore University, he was invited to a reception hosted by the Maharaja of Mysore that evening. The Maharaja wanted that all guests attending the reception would wear Western dress. Hearing this Asutosh was annoyed as he never worn Western dress and ordered his special railway carriage to be ready as he decided to leave Mysore that evening. As the news reached the Maharaja, he expressed his regret and extended warm welcome to Asutosh. Asutosh attended the ceremony proudly dressed in his Bengali dress.[3(ii)] From his very childhood Asutosh was an early riser and disciplined. 'His habits were most regular and he worked with clock-like precision'. Many visitors used to come to him at all hours into his house and his door was open to

all without any discrimination. All sorts of people came to him 'to seek advice and help from that man of stout heart and capacious brain.

There are many incidents to prove that Asutosh was a true friend and genuine well-wisher of students. His profound affection and his sincere concern for their true well-being drew many students spontaneously towards him for various purposes. Admst his multifarious and ever-increasing activities he was not only ready to listen to them but helped them with their requirements like books, money, counsel and guidance.

Though an orthodox Brahmin by birth, he was an ardent social reformer. In 1908 he gave his eldest daughter, who was a child widow, in marriage second time. His orthodoxy was an expression of his respect for the past and identity in the face of colonial rule. But he was not rigid. 'In his social outlook he resembled Iswar Chandra Vidyasagar, whom he must have regarded as his exemplar in many ways. That he had no religious fanaticism and that he was completely secular can easily be understood from the letter written on 23.01.1907 by Minto regarding appointing an Indian member of the Viceroy's executive council. In this letter we find, 'So far I have avoided naming any Indian as a possible member of my council, but if we decided to have one, the man whom I should name is Dr. Mookerjee, Vice-Chancellor of the University and High Court Judge. …… He is a Hindu but I doubt if his appointment would arouse Muhammedan criticism.'

Asutosh had a prodigious memory and a remarkable faculty of remembering faces and names of persons whom he had seen only once, and that year before. Also he could recapitulate everything in details of any incident once happened in front him long back. He had a huge collection of books on different subjects, but there was no catalogue. They lay scattered about in very possible corner of his house; but Asutosh could pick up the intended book at ease when he wanted as he knew where each book was to be found.

Narrating Asutosh's achievement H.J. Maynard justly said, 'His great voice and the ringing laugh, the long role of his eloquence, the bright and commanding eye, brought home to us the forces of that energy which conveyed such vitalizing influences to the Calcutta University.'[7]

Asutosh had a brilliant knack to identify talents and always helped and supported them for their full-fledged development. Appointment of professor like C. V. Raman, S. Radhakrishan and others [names of some of mentioned earlier] justify the claim.

Jyotiprasad Sarbadhikari uniquely described Asutosh: 'His versatile genius, keen intellect, encyclopedic knowledge, profound scholarship, legal learning, marvelous memory, extraordinary administrative and organizing power, quick appreciation of worth in others, and above all his fearless independence and intense patriotism and love of country with child-like simplicity of manners, an amiable, affectionate and kindly disposition made him a unique personality rare in any country.

Indeed, "The vision of Asutosh was boundless, his ability extraordinary and his courage earned him the popular title of 'Bengal Tiger'". He was absolutely fearless. He had a great self respect. As the Chancellor of the University when Lord Lytton offered him Vice-Chancellorship for another term after his fifth term under unjust conditions. The letter written to Asutosh was an insulting one. Asutosh not only refused the offer but gave fitting reply in his famous historical letter. In this letter he wrote, 'I quite realize that, I

have not in the remotest degree to please you, or your minister. But I claim that I have acted throughout, in the best interests of the University. I am not surprised that neither you, nor your minister can tolerate me. You have before one, who you can speak and act fearlessly according to his convictions, and you are not able to stand the sight of him.'[9(i)] He further wrote, 'I send you without hesitation only answer which an honourable man can send – an answer which you and your advisers expect and desire. I decline the insulting offer you have made to me.'

Another incident also narrates his stoutness of his character and boldness in his conviction. Lord Curzon invited Asutosh to go to England as the representative of Calcutta University to attend the coronation ceremony of King Edward VII. But Asutosh's mother strongly disapproved the idea. Asutosh conveyed to Lord Curzon his inability to go to England and its reason. Then Lord Curzon said, 'Please go and tell your mother that the Viceroy and the Governor General of India commands her son to go.' Asush replied, 'Then I must tell the Viceroy of India on her behalf that the mother of Asutosh Mookerjee refuses to let her son to be commanded by anybody excepting herself.

Asutosh was not only busy with study during his college days, but he was also involved actively in various activities. This is evident from the entries of his diary. Only selected portions of a few entries are given below to substantiate the claim:

(a) 'At 2 p.m., started to see the Convocation. Made way into the crowd, and got a good seat, the diplomas were distributed by the new Vice Chancellor, H.J.Reynolds. Two native ladies received their degree of B.A. This is the first instance in this University when the degree was conferred on them; there is a deafening roar of applause, claps.' (Entry on 10.03.1883)

(b) 'Went to see Cybeline in the Corinthian Theatre, where Pomeroy acted as Imogen. There were necessarily many excisions, though I must regret the omission of passages like the Soliloquy of Postham. The acting was really excellent, better beyond expectation. Imogen as Fidele was rendered very natural by Pomeroy's natural grace.' (Entry on February 3, 1883)

(c) 'The meeting of our club at 2 p.m. Satya delivered his address on moral greatness; he spoke slowly, and often in so low a tone that the delivery could easily be detected to be very artificial; there was a very hot discussion which Janaki Ram Bhatta of General Assembly Institution was allowed to take part; I also spoke for a while though frequently interrupted, and my speech was characterized by Mr. Percival as very fiery and eloquent; of course, it was radically extempore.' (Entry on April 14, 1883)

(d) 'Went to college at 10 a.m., but found that it was almost empty; almost everybody has gone to see Babu Surendra Nath Banerjee's trail at the High Court for an alleged contempt of Court, shown by something he had written in his paper "The Bengalee". The mob was so dense, that we could hardly get an entrance; as might have been expected, there was a quarrel with the police; the police was severely beaten, though four students were caught.' (Entry on May 4, 1883) [This incident had been especially mentioned by Surendra Nath Banerjee himself in 'A Nation in Making' : 'In the demonstration that followed the passing the sentence, the students took leading part, common among the young men all over the world, smashing windows and pelting the police with stones. One of these rowdy youths was Asutosh

Mukhejee, subsequently so well-known as a Judge of the High Court and as a Vice Chancellor of the Calcutta University.'[14(ii)] In this context Dinesh Chandra Sinha remarked that indeed this was first antigovernment movement in Bengal and that even against the judgment of H.C.]

(e) 'At noon, made an elaborate note book and began to write out my intended speech, to be delivered when Babu Surendra Nath Banerjee comes out of jail. In the afternoon went to see the prize-distribution of the Suburban School.' (Entry on May 13, 1883)

(f) 'Notices have been issued requesting students to gather near the gate of the jail to welcome Surendra Babu, when he comes up tomorrow, at day break. Came home at 4.30 p.m., made arrangements for carriages for taking us to the Presidency Jail tomorrow.' (Entry on July 3, 1883)

Patriotic Asutosh's love for freedom and voice against oppression can easily be observed from the following entries in his diary:

05.05.1883: 'Babu Surendra Nath Banerjee has been sentenced to two months' imprisonment—a most unjust thing; as Justice Mitter said, a fine of Rs. 500.00 would have been enough.

*I am fully disgusted with oppression and injustice of English rule in India.*Mind is gloomy'

07.05.1883: 'Read in the Statesman the judgment on Babu Surendra Nath. Grath's judgment is a decided failure. The language is very strong and the very tone of it shows that the case was got up from malice and the punishment , too, unnecessarily severe from malice.)

20.01.1884: 'Read the Statesman , the so-called Ilbert Bill will be brought before the Council next Friday, when undoubtedly it will be passed—poor India! My mother country, trodden down by tyrants—the worst tyrants, since they exercise the mild form of despotism, which is obviously thewrst form.'

02.01.1886: 'Burmah is annexed! I cannot find words to stigmatize the shameless action; Lord Dufferin has committed a crime for which he deserved to be hanged; one more nation is now reduced to status of slaves! Should any one clamour for any reasons against annexation, I would refer him to my Essay on the subject, where I have conclusively proved that it is a crime, not only a blunder.'

Regarding the famous personal library of Asutosh, it may be noted that Gangaprasad provided intellectual nourishment to his son by regular supply of valuable and necessary books from the beginning of Asutosh's school days. 'It is said that Sir Asutosh was presented with a fine collection of books worth about Rs.10,000. 00 then.'[18] This later formed the nucleus of his personal library. Asutosh also acquired books by various ways. Standing orders used to be placed with well known firms in Europe and America for supply of quality books. Also he collected many rare books from auction sales of private collection. From the following entries in his diary we may gather some idea about Asutosh's effort in procuring book to build his marvelous personal library:

20.01.1883: 'Received from Thacker a copy of Hudson's Shakespeare and Sir William Thomson's papers, volume 1; they are both capital books these, each in its department.'

8.01.1883: 'Wrote letters to W.J. C. Miller, Editor of the Educational Times, to Macmillan and Company ; to Hodgson and Son, but these were not posted, as stamps

could not had, the Post Office being closed.'

01.04.1883: 'Received from Hodgson & Son, Carr's Synopsis and 38th volume of Reprints from Educational Times. There were three problems and and solutions by me; but by a curious mistake these are all attributed to " Professor Mukhopadhyay, MA", as if I were an MA or a professor! Studied the solutions of some of the problems. At noon,studied Carr's Synopsis on Differential Calculus.'

12.04.1883: 'Received from Macmillan, the latest Cambridge Exam. Papers; they have arranged with the Masters of the Colleges to send those to me regularly, free of charge.'

16.08.1883: 'In the evening went to Auddy's book shop and bought my books, including, Laplace, Cauchy and Abel.'

17.10.1883: 'At 2p.m. set out from home, went to Thackers; no book could be got there; nxt to Auddy's; got Aryabata's Astronomy, German edition, and other books; came home at4.30 p.m.

[It may be an interesting observation that on 01.04.1883 Asutosh received the book 'A Synopsis of Elementary Results in Pure Mathematics' written by George Shoobridge Carr, popularly known as Carr's Synopsis. The book consisted of two volumes, first volume was published in 1880 and second volume was published in 1886. Naturally Asutosh received the first volume not the second volume. But Carr's Synopsis (consisting of precisely 4865 theorems and formulas on different branches of mathematics almost without proper proof) came to Indian mathematical wizard Srinivasa Ramanujan (1887–1920) sometime in 1903 and this event had been described as turning point of Ramanujan's life as this book 'first aroused Rumanian's full power' and 'through this the new world thus opened to him ; Ramanujan went ranging with delight.']

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A FEW QUTATIONS ABOUT SIR ASUTOSH

'His (Asutosh's) eminent success was due to his forceful character and extensive study, to his unremitting industry and brilliant advocacy, and to his genial temper and wonderfully quick comprehension.'—Syama Prasad Mookerjee [A Diary of Sir Asutosh Mookerjee; p.5]

'Sir Asutosh's boldness in vision and independence of thought "qualities absolutely necessary for success of a mathematical investigator "showed themselves very early when, as a student of first year class of Presidency College, he wrote his first paper in 1880'.

—Ganesh Prasad [Bull. of Cal. Math. Soc., Vol. XV (4), 1924-1925; p.53]

'Bengal in gaining a distinguished Judge and a great Vice-Chancellor lost in him (Asutosh) a still greater mathematician'.—C.V. Raman [Dinalipi (Bengali) by Asutosh adhyay; p.6]

'If he (Asutosh Mookerjee) had made up his mind to devote himself entirely to the study of mathematics, he was sure to have secured a place in the front rank of world mathematicians.'—Dr. R.P. Paranjpye [A Diary of Sir Asutosh Mookerjee; p.4]

"The vision of Asutosh was boundless, his ability extraordinary and his courage earned him the popular title of 'Bengal Tiger'."

—D.P. Sengupta [Current Science, Vol.78, No.12, 25 June, 2000; P.1566]

Mathematics For All—Teachers' Function to this Context

Dr. Mili Das*

Mathematics is an indispensible subject of study. It is difficult to carry on with our life or say daily life without the knowledge of mathematics or even can say basic mathematical knowledge and skills. Mathematics teaches us logic and order, help to develop power of reasoning and critical thinking among the learners.

We all know mathematics learning has manifold benefits. Still this subject is not at all enjoyable, interesting and inspiring to many of the young learners in the school levels. Many learners suffer from mathematics anxiety and are scared of the subject. Such learners naturally neglect mathematics which affects their performance and develop distaste in them.

These problems may be attributed to the ineffective ways of teaching mathematics as well as curriculum set up. Of course there are other reasons like parental pressure, inconvenience of family status and gender attitude etc., but here we are concentrating our discussion mainly for the students who are enrolled in primary, secondary and higher secondary sections.

As we think about only school going children we must look at the vision and mission set by the NCF-2005, which are as follows

Vision for School Mathematics: [According to National curriculum Frame Work- 2005 by NCERT]:

- Children learn to enjoy mathematics rather than fear it.
- Children learn important mathematics: mathematics is more than formulas and mechanical procedures.
- Children see mathematics as something to talk about, to communicate through, to discuss among them, to work together on.
- Children pose and solve meaningful problems.
- Children use abstractions to perceive relation-ship, to see structures, to reason out things, to argue the truth or falsity of statement.
- Children understand the basic structure of mathematics: Arithmetic, Algebra, Geometry and Trigonometry, basic content areas of school mathematics, all offer a methodology for abstraction, to develop structure and generalization.
- Teachers engage every child in class with the conviction that every child can learn mathematics.

Discussion

Being a part of the present system of education we the mathematics teachers cannot claim that we are successful to prioritize 'Mathematics for All" in our society. Then automatically the question arises what are the reasons and defects in learning mathematics?? Why distaste in learning mathematics spatially in the school students are widely spreading.

*Former Jt. Director (Trg.), Govt. of W.B

Even now a day's many students who are doing excellent in literatures and social sciences have the opinion that mathematics is a boring subject. Let us find the real situation. In every public and private sphere mathematical knowledge and its applications have become indispensable tools, still…

In our primary and secondary system mathematics is a compulsory subject. But more than 50% students are afraid of learning mathematics and the mathematics teacher. Many of these students do not feel ashamed to say I am very weak in mathematics or I hardly pass in mathematics. It is found that students get anxious before attending mathematics classes; the following area may cause the fact.

1. In appropriate teaching and learning system
2. Environment-not suitable
3. External and Internal pressure

■ **In appropriate teaching and learning system—why?**

(a) Inappropriate Syllabus which in turn creates problems in the process of teaching, as it is mentioned in 2nd point of vision that children don't want scare about topics and those must be interesting to work with; not to remember formulae and mechanical practice of doing sums.

(b) *No scope of Review/lack of Correlation:* Children like to compare, develop the concept of abstractness in properties and generalize, from the structural approach as it is cited in the vision of NCF.

(c) *No scope of revisit/revision:* Teachers hardly recapitulate and reinforce during class teaching.

(d) *Not written in the friendly language:* Text Books developed and prescribed in the school levels are improved in presentation but the diagrams, language and overall sequential arrangement of topics are not yet logical always to students, sometimes they don't get the reason in some topics.

(e) *Does not challenge:* In general govt. and govt. aided schools the students in a class are usually heterogeneous but sums included i.e. workout examples and exercises in the mathematics text books are set mechanically without considering the real situation of the classes;

(f) *Evaluation:* In general mathematics teachers emphasize on the correctness of the product but not the process which includes mathematical language, understanding, approach and application in development of content. In fact, Mathematics learning and achievement are a function of three factors, related to students, content and teaching models. It is the interaction of three sets of variables: Content, learners and teaching. In evaluation process the three variables are needed to be specifically analyzed and judged, and then the process will be scientific. To evaluate students' level of achievement, teacher must pay attention to:

1. Potential for learning mathematics,
2. Mathematics learning personality,
3. Preparation for learning mathematics,
4. Development of Mathematics Language,

5. Level of Mathematics achievement and mastery,
6. Approach, attitude towards mathematics learning and problem solving.

■ **Incompetence of Teachers directly affect learning situation as**

(a) Teachers lack of knowledge make the methodology of teaching vague & unclear: As mentioned in the vision of NCF-2005 mathematics teachers will take care of every student during class teaching but he/she is not confident in selecting and applying a method and techniques for a particular topic to instruct ; in that situation teacher himself or herself become puzzled;

(b) Do not prepare lesson that makes teaching haphazard;

(c) Give too much home-work leads to non participation and alienates learners.

■ **Lack of Resources affect regular teaching learning system as it includes**

(a) Human resource that includes trained teachers and family support

(b) Appropriate and adequate text and references: Teachers hardly refer reference books and do not help the learner in using the same.

(c) School library many times do not have sufficient reference books and journals on mathematics that includes variety of items like auto biography of famous mathematicians, about discoveries, puzzles in mathematics and current invention etc.

2. Environmental factors that increase negative feeling to learn mathematics are—

(a) Lack of Academic Experiences: May be first generation learner where regular attendance hampered and cannot follow teacher and sequence of the teaching.

(b) Negative family tradition: Sometimes Girl-children are neglected or parents have negative feeling regarding learning of mathematics, as no one of the family has got success in learning the subject and many others.

(c) Inaccessibility of Teachers: As the subject is not popular to many students automatically there grow a fear about the mathematics teachers. On the other hand shortage of mathematics teachers in the schools that create over burden on the said teachers and obviously they do not get time to meet their students.

3. External and Internal pressure:

(a) Social: i.e. Pal pressure means pear group's, friends' & neighbors' comment. Sometimes senior relatives or similar persons compare and advice that the' subject is not for the child etc.

(b) Family: Parents aspiration, other members of the family expect high score.

(c) Finance: Big family, Duties in the family, lack of parenthood, outgoing parents

Information:

In the above discussion I mentioned about the family support in making mathematics learning easier. In our country in case of education family support means either monetary help or the strength of manpower, but not in general family as a tutor or a complementary part of mathematics teaching-learning? But in developed countries like USA, UK and many other countries reported by UNESCO, the educators are thinking and experimenting about how the concept 'Mathematics For All 'can be popularized and accepted by general people. For example:

The Harvard School of Education in USA, they are interested not only in Mathematics for All but Mathematics, Everywhere for Every one'. The proposed approach is to bridge up the gap between math in classroom and math at home- for all families. And the new resources from the 'Harvard Family Research Project' (HFRP) paint a different picture that the role of the family as a source of mathematics knowledge where the role of the family as a source of math knowledge. The assumption is that students learn best when their families and teachers are co-learners.

'The Mathematics Faculty, University of Cambridge offers a free online Outreach program of Mathematics for schools to work with students, teachers, families and general public through a range of innovative activities and events. Their motto is to encouraging, inspiring imaginative mathematics for all. Also the Faculty's 'Millennium Mathematics Project' (MMP) is a pioneering collaboration with the Faculty of Education. The MMP consists of a family of award winning outreach and education activities, including the very successful NRICH and Plus websites, and face to face work with students, teachers and the general public.'….

Conclusion :

The Secondary Education Commission appointed in 1952 and emphasized the need for mathematics as a compulsory subject in the schools. The National Curriculum Framework for School Education (NCFSE) 2000 have identified the problem 'why mathematics is not a popular subject' to students and The National Curriculum Framework 2005 has framed their vision for school education, 'what measure should be taken' to make mathematics a subject of interest to students, teachers and general public. But to make the current theme 'MATHE MATICS FOR ALL 'more effective widen approach to be taken up so that a huge number of people get scope to participate in the project.

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Martyr Women Astronomer, Mathematician Hypatia

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Hypatia a neoplatonist philosopher, astronomer, and mathematician, was born c. 350–370 and died 415 AD lived in Alexandria, Egypt, then part of the Eastern Roman Empire. Hypatia's exact year of birth is still under debate, with suggested dates ranging from 350 to 370 AD. Many scholars have followed Richard Hoche in inferring that Hypatia was born around 370. According to a description of Hypatia from the lost work *Life of Isidore* by the Neoplatonist historian Damascius, preserved in the entry for her in the *Suda*, a tenth-century Byzantine encyclopedia, Hypatia flourished during the reign of Arcadius. Hoche reasoned that Damascius's description of her physical beauty would imply that she was at most 30 at that time, and the year 370 was 30 years prior to the midpoint of Arcadius's reign. In contrast, theories that she was born as early as 350 are based on the wording of the chronicler John Malalas, who calls her old at the time of her death in 415. Robert Penella argues that both theories are weakly based, and that her birth date should be left unspecified.

Hypatia was a prominent thinker in Alexandria where she taught philosophy and astronomy. Although preceded by Pandrosion, another Alexandrine female mathematian, she is the first female mathematician; whose life is reasonably well recorded. Hypatia was renowned in her own lifetime as a great teacher and a wise counselor. She wrote a commentary on Diophantus's thirteen-volume *Arithmetica*, which may survive in part, having been interpolated into Diophantus's original text, and another commentary on Apollonius of Perga's treatise on conic sections, which has not survived. Many modern scholars also believe that Hypatia may have edited the surviving text of Ptolemy's *Almagest*, based on the title of her father Theon's commentary on Book III of the *Almagest*.

Hypatia's father Theon of Alexandria is best known for having edited the existing text of Euclid's *Elements*. Theon was the head of a school called the "Mouseion", which was named in emulation of the Hellenistic Mouseion, whose membership had ceased in the 260s AD. Theon's school was exclusive, highly prestigious, and doctrinally conservative. Theon rejected the teachings of Iamblichus and may have taken pride in teaching a pure, Plotinian Neoplatonism. His primary achievement was the production of a new edition of Euclid's *Elements*, in which he corrected scribal errors that had been made over the course of nearly 700 years of copying. Theon's edition of Euclid's *Elements* became the most widely used edition of the textbook for centuries and almost totally supplanted all other editions. In short her contribution may be classified in following way:

1. **Commentary on Ptolemy's Almagest**

   It was believed that Hypatia had only revised the commentary made by his father on Ptolemy's Almagest, titled "Commentary by Theon of Alexandria on Book III of Ptolemy's Almagest". But some of the scholars claimed that Hypatia did not revise his father's commentaries, instead, she corrected the text of Almagest. Hypatia contributed by providing a better method of long division that is used in astronomical computations. Also, Ptolemy in his book mentioned a division

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problem to calculate the number of degrees covered by the sun in a single day as it orbits the earth. In the first edition, Theon tried to improve the calculation done by Ptolemy. In Hypatia's edition, she demonstrated a tabular method for the calculation. A classicist, Alan Cameron claimed that it might be a possibility that Hypatia not only edited book III but also the rest nine extant of book Almagest.

2. **Commentary on Diophantus's Arithmetica**

   Hypatia wrote a commentary on Diophantus's Arithmetica that contained 100 mathematical problems solved using algebra. It was believed that only volumes I-VI of Arithmetica had survived and the rest are lost, but it is later found that four additional volumes were preserved in Arabic translation. This Arabic translation contained some additional problems and verification of Diophantus examples that were not present in the Greek text. Paul Tannery, a scholar of the 19th century was the first to deduce that the additional material in the Arabic translation of Arithmetica was added by Hypatia. Cameron also claimed the same and said that the additional work on Arithmetica followed the same pattern as used by her father, Theon. Heath after publishing the first English translation of the surviving extents of Arithmetica claimed that these texts were produced by Hypatia to help her school students. Wilbur Knorr, a historian of mathematics argued that the additions are "of such low level as not to require any real mathematical insight", and that does not go well with Hypatia's high abilities as a philosopher and mathematician. Cameron rejects this argument by saying that this edition of Hypatia's work was specially designed for students rather than for professional use.

3. **Astrolabe**

   One of the letters of Synesius revealed that Hypatia taught him how to construct an Astrolabe-inclinometer used by past astronomers to predict local time by locating the positions of the stars, sun, and moon. Hypatia might have learnt how to construct an astrolabe from her father, as he wrote two books titled, Memoirs on the Little Astrolabe and another study on the armillary sphere in Ptolemy's Almagest. Hypatia and her father must have studied Ptolemy's Planisphaerium that contained necessary calculations for the construction of an astrolabe.

4. **Hydrometer**

   Hypatia was credited for inventing a device known as a hydrometer as there was evidence of a letter in which Synesius asked Hypatia to construct a hydrometer for him. This device is used to determine the density of fluids. It is based on Archimedes 3rd century principle and some believed that it might be invented by him. In the letter, Synesius mentioned every minute detail of the hydrometer that shows, Hypatia hardly had any idea about this device but Synesius believed that she would be able to construct it with the help of given details.

5. **Lost works**
   - A commentary on Archimedes's Sphere and Cylinder surviving as John of Tynemouth's De Curvis Superficibus, a text on isoperimetric figures incorporated by a later author into Introduction to the Almagest.

- A commentary on Archimedes's Dimension of the Circle.
- A commentary edition of Apollonius Pergaeus's Conics upon which later commentary editions were based.
- She also created an "Astronomical Canon", which is believed to have been either a new edition of the Handy Tables by Alexandrian Ptolemy or a commentary on his book Almagest.

Although she herself was a pagan, she was tolerant towards Christians and taught many Christian students, including Synesius, the future bishop of Ptolemais. Ancient sources record that Hypatia was widely beloved by pagans and Christians alike and that she established great influence with the political elite in Alexandria. Towards the end of her life, Hypatia advised Orestes, the Roman prefect of Alexandria, who was in the midst of a political feud with Cyril, the bishop of Alexandria. Rumors spread accusing her of preventing Orestes from reconciling with Cyril and, in March 415 AD, she was murdered by a mob of Christians led by a lector named Peter.

Death of Hypatia

From 382–412, the bishop of Alexandria was Theophilus. Theophilus was militantly opposed to Iamblichean Neoplatonism and in 391, he demolished the Serapeum. Despite this, Theophilus tolerated Hypatia's school and seems to have regarded Hypatia as his ally. Theophilus supported the bishopric of Hypatia's pupil Synesius, who describes Theophilus in his letters with love and admiration. Theophilus also permitted Hypatia herself to establish close relationships with the Roman prefects and other prominent political leaders. Partly as a result of Theophilus's tolerance, Hypatia became extremely popular with the people of Alexandria and exerted profound political influence.

Theophilus died unexpectedly in 412. He had been training his nephew Cyril, but had not officially named him as his successor. A violent power struggle over the diocese broke out between Cyril and his rival Timothy. Cyril won and immediately began to punish those who had supported Timothy; he closed the churches of the Novatianists, who had supported Timothy, and confiscated their property. Hypatia's school seems to have immediately taken a strong distrust towards the new bishop, as evidenced by the fact that, in all his vast correspondences, Synesius only ever wrote one letter to Cyril, in which he treats the younger bishop as inexperienced and misguided. In a letter written to Hypatia in 413, Synesius requests her to intercede on behalf of two individuals impacted by the ongoing civil strife in Alexandria, insisting, "You always have power, and you can bring about good by using that power." He also reminds her that she had taught him that a Neoplatonic philosopher must introduce the highest moral standards to political life and act for the benefit of their fellow citizens.

According to Socrates Scholasticus, in 414, following an exchange of hostilities and a Jewish-led massacre, Cyril closed all the synagogues in Alexandria, confiscated all the property belonging to the Jews, and expelled a number of Jews from the city; Scholasticus suggests all the Jews were expelled, while John of Nikiu notes it was only those involved in the massacre. Orestes, the Roman prefect of Alexandria, who was also a close friend of Hypatia and a recent convert to Christianity, was outraged by Cyril's actions and sent a scathing report to the emperor. The conflict escalated and a riot broke out in which the *parabalani*, a group of Christian clerics under Cyril's authority, nearly killed Orestes. As punishment,

Orestes had Ammonius, the monk who had started the riot, publicly tortured to death. Cyril tried to proclaim Ammonius a martyr, but Christians in Alexandria were disgusted, since Ammonius had been killed for inciting a riot and attempting to murder the governor, not for his faith. Prominent Alexandrian Christians intervened and forced Cyril to drop the matter. Nonetheless, Cyril's feud with Orestes continued. Orestes frequently consulted Hypatia for advice because she was well-liked among both pagans and Christians alike, she had not been involved in any previous stages of the conflict, and she had an impeccable reputation as a wise counselor.

Despite Hypatia's popularity, Cyril and his allies attempted to discredit her and undermine her reputation. Socrates Scholasticus mentions rumors accusing Hypatia of preventing Orestes from reconciling with Cyril. Traces of other rumors that spread among the Christian populace of Alexandria may be found in the writings of the seventh-century Egyptian Coptic bishop John of Nikiû, who alleges in his *Chronicle* that Hypatia had engaged in satanic practices and had intentionally hampered the church's influence over Orestes.

Murder

According to Socrates Scholasticus, during the Christian season of Lent in March 415, a mob of Christians under the leadership of a lector named Peter, raided Hypatia's carriage as she was travelling home. They dragged her into a building known as the *Kaisarion*, a former pagan temple and center of the Roman imperial cult in Alexandria that had been converted into a Christian church. There, the mob stripped Hypatia naked and murdered her using *ostraka*, which can either be translated as "roof tiles" or "oyster shells". Damascius adds that they also cut out her eyeballs. They tore her body into pieces and dragged her limbs through the town to a place called Cinarion, where they set them on fire. According to Watts, this was in line with the traditional manner in which Alexandrians carried the bodies of the "vilest criminals" outside the city limits to cremate them as a way of symbolically purifying the city. Although Socrates Scholasticus never explicitly identifies Hypatia's murderers, they are commonly assumed to have been members of the *parabalani*. Christopher Haas disputes this identification, arguing that the murderers were more likely "a crowd of Alexandrian laymen".

Socrates Scholasticus presents Hypatia's murder as entirely politically motivated and makes no mention of any role that Hypatia's paganism might have played in her death. Instead, he reasons that "she fells a victim to the political jealousy which at that time prevailed. As she had frequent interviews with Orestes, it was calumniously reported among the Christian populace that it was she who prevented Orestes from being reconciled to the bishop." Socrates Scholasticus unequivocally condemns the actions of the mob, declaring, "Surely nothing can be farther from the spirit of Christianity than the allowance of massacres, fights, and transactions of that sort."

The Canadian mathematician Ari Belenkiy has argued that Hypatia may have been involved in a controversy over the date of the Christian holiday of Easter 417 and that she was killed on the vernal equinox while making astronomical observations. Classical scholars Alan Cameron and Edward J. Watts both dismiss this hypothesis, noting that there is absolutely no evidence in any ancient text to support any part of the hypothesis. ***Women Astronomer, Mathematician Hypatia was murdered by religious fanatic people***.

Aftermath

Hypatia's death sent shockwaves throughout the empire; for centuries, philosophers had been seen as effectively untouchable during the displays of public violence that sometimes occurred in Roman cities and the murder of a female philosopher at the hand of a mob was seen as "profoundly dangerous and destabilizing". Although no concrete evidence was ever discovered definitively linking Cyril to the murder of Hypatia, it was widely believed that he had ordered it. Even if Cyril had not directly ordered the murder himself, it was self-evident that his smear campaign against Hypatia had inspired it. The Alexandrian council was alarmed at Cyril's conduct and sent an embassy to Constantinople. Theodosius II's advisors launched an investigation to determine Cyril's role in the murder.

The investigation resulted in the emperors Honorius and Theodosius II issuing an edict in autumn of 416, which attempted to remove the *parabalani* from Cyril's power and instead place them under the authority of Orestes. The edict restricted the parabalani from attending "any public spectacle whatever" or entering "the meeting place of a municipal council or a courtroom." It also severely restricted their recruitment by limiting the total number of parabalani to no more than five hundred. Cyril himself allegedly only managed to escape even more serious punishment by bribing one of Theodosius II's officials. Watts argues that Hypatia's murder was the turning point in Cyril's fight to gain political control of Alexandria. Hypatia had been the linchpin holding Orestes's opposition against Cyril together, and, without her, the opposition quickly collapsed. Two years later, Cyril overturned the law placing the *parabalani* under Orestes's control and, by the early 420s, Cyril had come to dominate the Alexandrian council.

Hypatia's murder shocked the empire and transformed her into a "**martyr for philosophy**", leading future Neoplatonists such as Damascius to become increasingly fervent in their opposition to Christianity. During the Middle Ages, Hypatia was co-opted as a symbol of Christian virtue and scholars believe she was part of the basis for the legend of Saint Catherine of Alexandria. During the Age of Enlightenment, she became a symbol of opposition to Catholicism. In the nineteenth century, European literature, especially Charles Kingsley's 1853 novel *Hypatia*, romanticized her as "the last of the Hellenes". In the twentieth century, Hypatia became seen as an icon for women's rights and a precursor to the feminist movement. Since the late twentieth century, some portrayals have associated Hypatia's death with the destruction of the Library of Alexandria, despite the historical fact that the library no longer existed during Hypatia's lifetime.

Some Reflections of NEP-2020 on School Math Curriculum.

Prof. M. M. Chel*

Introduction:

National Education Policy-2020 (NEP-2020) has restructured school education as follows.

Stage	Duration	Age of Learner	Classes
Foundation stage	5 years	3-8 yrs	3yrs Angunwadi/preschool
Preparatory stage	3 years	8-11 yrs8-11 yrs	2 yrs in primary G1 + G2 Preparatory stage G3, G4, G5
Middle stage	3 years	11-14 yrs	G6, G7 and G8
Secondary stage	4 years	14-18 years	G9, G10, G11 and G12

The preschool stage *i.e.*, Early Childhood Care and Education (ECCE) will be the foundation of learning. NEP has emhasized outcome based curriculum, vocationalization, experiental learning, practical oriented education system. The main focus is to increase students learning rathar mare teaching the syllabus of any subject. Students need to taught about "How to think rather what to think." Pedagogy in school will be such that learning should be Holistic, Integrated, Enjoyable and Engaging. The concepts that have been incorporated in NEP regarding school curriculum and pedagogy are not new but NEP has specified and has given more stress. 'Outcome' based curriculum for a particular class means before teaching the content both the teachers and students have some expected outcomes in the knowledge domain as well as in behavioural changes domain. For example when the students will be exposed to the concept of 'interest' in mathematics, the expected outcomes are (a) students will understand the mathematical concept of interest, (b) they will be able to calculate the interest from the verbal problems, (c) if they found the rate of interest in govermental organisation is less than non-governmental or private organisations then they will motivate their parents to take loan from banks and invest money in private organisation for better benefits provided the private organisations are reliable and trusteworthy. Similarly, 'experiental learning' intends to offer scope for verification through hands-on-experience of the theory and concepts taught in the class rooms. In mathematics, mathematical concepts are limited but their application are many and varried. These application of a particular mathematical concept may be set in different classes incorporating the difficulty values of the applications. For example, the concept of 'unitary method' is to be discussed as a whole, i.e., holistically then students will be understandable that 'profit and loss' or 'interest' are mere extended applications of 'unitary method'. For integration of a topic of mathematics in a particular class, we can integrate with other topics of mathematics and can integrate with other science subjects. For example, concept of 'variation' can be integrated with charle's and Boyle's laws. For making mathematics enjoyable and engaging we can arrange 'Mathematics for fun' in which different activities are to be performed by the students to

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verify the rules and formulae of mathematics. It is obvious higher the time of engagement with mathematical activities of the learner better will be the performance level of students.

Inclusion of New Concepts in School Mathematics:

NEP has recognised that mathematics and mathematical thinking will be very important for India's future and India's forwardness role in the various upcoming fields and profession. The concepts of artificial intelligence, machine learning, data science and coding activities are included in mathematics curriculum. Mathematics and Computational Thinking will be given more emphasis throughout the school years starting with fundamental through a variety of activities like puzzles and games. Fun course will be introduced during Grade 6-8 which will include survey, hands-on experience of sampling, coding activities etc. '10 days bagless period' programme will include different enrichment activities like mathematical quiz, quests, mathematics take and application of mathematical concepts taught in a particular class in other fields or in daily life situations.

According to NEP-2020 (4.5) curriculum content will be reduced to its core essential to make space for critical thinking, inquiry based, discovery based, discussion based and analysis based. Classroom activities will contain more fun, creative, collaborative and exploratory activities for students. To reduce the weight of school bags and text book, the syllabus will contain only core materials. Subject mathematics has some unique characteristics. Here the contents are to be included in a graded manner. So to reduce the existing contents and to incorporate these mathematical concepts is a difficult job. But state government can not deny the recommendations. So time has come to take judicious decission inconsultation with the experts bodies. National curriculum Frame Work for School Education (NCFSE) 2020-2021 has already framed the draft syllabus for implementation.

During eighties, a group of resource persons of Centre for Pedagogical Studies in Mathematics (CPSM) had contributed towards the framming of mathematics Text books (class VI-X) of WBBSE (West Bengal Board of Secondary Education), WBMB (West Bengal Madrash Board) and Rabindra Mukta Vidyalaya.

Discussion on New Mathematical Concept to be Included:

A. Artificial Intelligence (AI):

It refers to the simulation of human intelligene in machine to include learning, reasoning and perception using algorithms. Here intelligence is demonstrated by machine as opposed to the natural intelligence displayed by human and animals. Algorithms play a very important part in the structure of AI. AI is a field which combines mathematics, computer science, linguistics, psychology etc. and huge data sets etc.

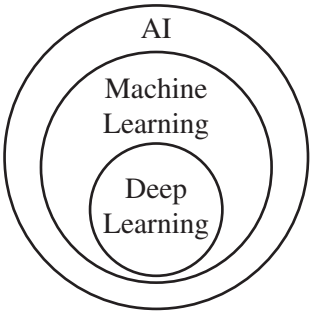
Types of Aritficial Intelligence (AI):

(i) Weak/narrow or single task oriented AI which is focused to perform specific task.

(ii) Strong AI is made up of artificial general intelligence and focussed to perform more complex task human like.

AI applications include advance web search engine like Google, recommendation system used in YouTube, Amazon, understanding human speech, self driving cars, automated decision making like strategic game system used in chess etc.

AI includes subfields of machine learning and deep learning.



B. Machine Learning (ML):

It is an application of Artificial Intelligence (AI) that enables software application to become more accurate at predicting outcomes without being explicity programmed to do so. ML allows the computer to learn the input (*i.e.*, data or examples provided) autonomously without human intervention or assistance and adjust actions to look for pattern in data and finally make inference based on data or examples provided.

The ML process can be traced as follows:

Step I : Identification of relevent data set/examples and to prepare them for analysis.

Step II : To select the types of ML algorithm to be used.

Step III : To build an analytical model based on the selected algorithm.

Step IV : To train the model on text data sets and to revise if needed

Step V : To run the model to generate findings as inference.

So a part of ML acts as subfield of AI or a part of AI acts as subfield of ML (Machine Learning). Arthur Samuel first applied the concept of ML in playing cheekers.

Now ML is used in self learning car, automatic language translation, automatic sentiment analysis, fraud detection, in games like chess, checkers, e-mail filtering, predictive analysis in business process etc. Here data is key and Artificial Intelligence (AI) is the goal.

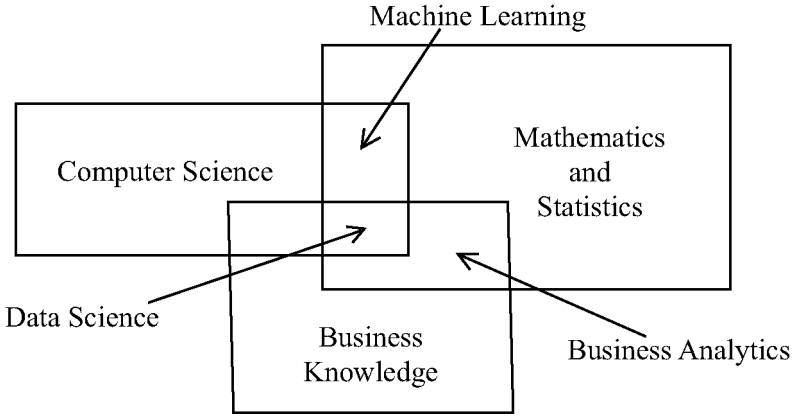
C. Data Science:

It is a blend of various tools, algorithms and machine learning principles to discover the hidden pattern form structural and unstructural data. Data science is an interdisciplinary field of computer science, mathematics and business knowledge. Data science uses mathematics, statistics extensively along with machine learning algorithms to design statistical modes to develop knowledge from the pile of big data known as data mining. Big data is the scientific techniques to process huge volume of data to extract information using mathematical analysis,

optimization and statistics. While data science interprets the data and extracts the information using techniques and methods to evaluate the potential of big data in a precise way.

Data science can be represented in the following Venn diagram:

Venn diagram shows the similarities and difference between computer.



The knowledge of the topics of statistics and mathematics that will develop the essential skill of data science are the following:

Topics of statistics:

(a) Mean (b) Median (c) Mode

(d) Standard deviation (e) Correlation coefficient (f) Probability distribution

Most of the topics exist in our secondary syllabus and a few in HS syllabus.

Topics of mathematics:

Algebra: (a) vectors, (b) matrixes, (c) Transpose of a matrix, (d) The inverse of a matrix, (e) The determinant of a matrix, (f) Dot product etc.

Calculas: (a) Function of several variables, (b) Derivatives and gradients, (c) Minimum and Maximum values of a function, (d) Ploting of functions etc.

Linear algebra is used in data pre processing and data transformation. The term data analytics means an analysis of a particular point in time say past while data science takes into account the past as well as the present time data to predict the future events.

D. Coding Activities:

Coding is a method of communicating with a computer. It is an act of translation. It writes the user's instruction in the form of computer programme. Computers do not understand human language. Coding helps human to communicate with computers. It is used for writing systematic lines of instructions to perform certain actions. So codes tell a computer what action to take and writing code is like creating a set of instructions. The binary code represents 'on and off' transistors as 1 and 0 digits. An infinite number of combinations of these codes make computer work. To make binary code manageable, computer programming language are formed.

Coding is mathematics and mathematics is coding:

Logical thinking and computational skill are the basis of mathematics. On the other hand coding requires algorithmic thinking (based on logical sequencial pattern of thinking) and computational thinking. So coding, at the bottom line is mathematics when we teach the students mensuration, we give mathematical idea how to calculate area or perimeter of a garden or for calculating the number of tiles will be required to cover the base of a floor etc. Using coding activities, the learner will be amazed by observing the floor space covered with tiles. The calculated value will be verified by coding techniques. Thus the abstract nautre of mathematics will be joyful to learner. Using coding activities students will be able to find out the relationship between or among variables using diagrams, graphs and flow charts. Coding enhances problems solving skills and abstruct reasoning of the students.

Coding may be used as a way to model mathematics:

Coments:

For inclusion of such concepts in the form of mathematical topics adequate arrangement like availability of personal computer/tabs or atleast well equipped computer laboratory in each school, qualified teachers are to be assured. We have discussed the theoretical frame work of those new concepts. More homework are to be done before finalisation of the inclusion of those concepts. Discussions, deliberations and debates among the teachers, experts will definitely enhance the quality of mathematics syllabus. Change is an asset. It is not something to be afraid of.

Conclusion:

Our intension is to make our mathematics syllabus more updated and acceptable to all *i.e.*, students and teachers up to the level of higher secondary. To infuse the concepts of computer science in school mathematics, we may remember the saying "The best and worst thing about computer is that the computer will do exactly what you tell it to do."

Early Identification and Interventions for Dyscalculia

Abdus Safi*

Abstract

Approximately 6 percent of the population has dyscalculia, a specific learning problem. Although children with dyscalculia have difficulty learningmathematics and have trouble understanding numbers intuitively, they are not unintelligent and may do really well in non-mathematical disciplines.But this is a learning difficulty that is often not noticed. The aims of this paper are to search at studies and other related literature about the idea of dyscalculia, the different kinds of dyscalculia, how to diagnose and treat dyscalculia, and how dyscalculia is related to dyslexia, and how dyscalculics and normal children are alike. When studies are looked at, it becomes clear that less research has been done on math problems. More research needs to be done on dyscalculia if the problems that come with it are to be lessened.

• **KeyWord :** Dyscalculia, Dyslexia, Diagnostictest

Introduction

Math is a subject that requires many different skills, like being able to count and figure out numbers. If children don't know how to do the basics of math, it might be hard for them to move on to more complicated math applications. The poor quality of the education at primary level stages gives children a weak foundation and it directly impacts their learning at the high school stage and even carries over even more. Math is hard for a lot of children, not be cause they are dumb, but because they don't know how to do it. Learning achievement surveys recently done by the National Council of Educational Research and Training (NCERT) show that math classes need more attention to help children learn the basics of math. So, teaching math to children has be come harder[20]. According to Reeve (2011), dyscalculiais as follows: *"It is often co-morbid with other problems such as dyslexia; sometimes it is very different from dyslexia.*

I think that the general agreement now is that there is a pure form that is not associated with other learning difficulties.We suspect that it is genetic, or at least a neurological problem in some respects."[11]

How common is Dyscalculia?

Dyscalculia affects about 5 percent of school-age children.According to Sue Gif-ford' sreviewoftheresearch,'Young Children's DifficultiesinLearningMathematics,' although findings of prevalence studies ranged between 3 and 6 percent, Shalev et al. (2000) 'concluded that a realistic estimate was 5 percent, as with dyslexia [22]. This is supported by numerous current sources Gifford (2005) [7] Because different criteria for diagnosis are used, researchers have found it difficult to determine how many people suffer from dyscalculia (Wilson2010) [28].

Istherea Cure?

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Because dyscalculia is not a disease, there is no cure. Many dyscalculics can become competent mathematicians if they are taught correctly using organized, multi-sensory teaching methods like the one developed at Emerson House (Kay and Yeo 2003, Yeo 2003, Butterworth and Yeo 2004) [29, 2].

Types of Dyscalculia

Children with Developmental Dyscalculia (DD) can fail at a wide range of numerical tasks, such as performing arithmetic operations, solving arithmetic problems, as well as using numerical reasoning. Kosc (1970) described six types of difficulties observed in DD: "(1) problems in verbal organization of numbers and mathematical procedures (2) difficulties in the management of mathematical symbols or objects (3) errors in reading numbers (4) errors in writing numbers (5) difficulties in the understanding of mathematical ideas, and (6) in the carrying over when performing arithmetical operations.Students with dyscalculia may present a cluster of problems in their ability to perform mathematical tasks" [13, 1]. According to Strang Rourke (1985), the errors found in children with dyscalculia can be classified into seven categories: (1) errors in spatial organization of quantities, (2) errors in visual attention, (3) arithmetical procedural errors, (4) graphic motor errors when writing quantities, (5) numerical judgment and reasoning errors, (6) memory errors for quantities, and (7) preservation in solving arithmetical operations and numerical problems. Table-1 describes the characteristics of the most frequent types of errors found in children with DD.

Spatial

Difficulties in placing numbers in columns, following appropriate directionality of the

procedure,v.gr.,	to subtract the sub strand from the minuend Visual Difficulties in reading arithmetic signs, forgetting the points of the units
of thousand,	etc. Procedural
application of	Omission or addition of a step of the arithmetical procedure, and a learned rule for a procedure to a different one, v.gr., 75 + 8 = 163, an operation in which the multiplication rule is appliedin the sum Graphomotor Judgement
Difficulty in forming the	appropriate numbers. Errors that imply impossible results, such as one in which there sult of subtracting is bigger than the numbers being subtracted
Memory	Problems in the recall of multiplication tables or arithmetical procedures
Perseveration	changing from one task to another one repetition of the same number
Difficulty in	

Table-1: Error Most frequently Found in Children with DD Adapted from Strang Rourke (1985) [1]

Strang and Rourke (1985) worked together to show that children with dyscalculia

have significant problems with their visual-perceptual organization. They also wrote about problems with other neuropsychological tasks. These children had trouble figuring out what things were by touching them, especially with their left hands.They also had trouble understanding facial and emotional expressions [24]. Rosenberger (1989) found that children who had trouble with math were more likely to have problems with how they visualize things and how they paid attention.He also revealed that children who have trouble with math have more obvious problems with visuospatial and attention dysfunctions [17]. Geary (1993) tried to divide dyscalculia into three groups based on the three kinds of mistakes that people with the disorder make : (1) visual-spatial, (2) semantic memory, and (3) procedural [6]. Rourke (1993) has identified two types of developmental dyscalculia: 1) dyscalculia linked to language problems (dyslexia), problems understanding instructions and verbal problems, and a lower capacityfor verbal memory; and 2) dyscalculia linked to problems with sequences and reversing numbers (Spreen et al, 1995)[24, 23].

Diagnosis

A child's arithmetic skills are assessed to determine if he or she has developmental dyscalculia. Reynolds(1984) and Semrud-Clikeman (1992) say that a child's assessment is based on the difference between his or her intellectual potential and arithmetic achievement, or on a difference of at least two years between the child's chronological grade and the level of achievement [16, 19]. Mc-Closkey et.al. (1985) developed and validated an arithmetic test based on the neurocognitive model of McCloskey. According to Jastak (1984) standardized arithmetic tests are the acceptable method to assess arithmetic skills in children. Timed tests that emphasize arithmetic achievement include the WART-R [12] arithmetic subtests and Young's collection of mathematics tests [21]. A battery of arithmetic tests for children called the Neuropsychological Test Battery for Number Processing and Calculation was developed recently by Swets Test Services (2000)[26].Emerson et al.(2014), developed dyscalculia assessment on number sense, counting, calculation, place value, multiplication, division, word problems, and formal written numeracy.He also gave general instruction for conducting the assessment to find dyscalculia among primary school children [5]. Chinn (2020), developed a Dyscalculia checklist from which we can identify 31 characteristics that can contribute to maths failure.He constructed a 15-minute norm-referenced mathematical test to identify different mathematical learning disabilities and error patterns[3].

Remedial Programor Treatment

When practical, Strang and Rourke (1985) recommend including systematic and concrete verbalizations of operations and arithmetical procedures in remedial programmes for children with dyscalculia. Mechanical arithmetic procedures should be transformed into verbal assignments that allow the child to dissect the operations and therefore enhance his or her learning. Clear, explicit, accurate,and systematic teaching methods should be used [24]. Sullivan (1996) designed a training program for a child who had difficulty transcoding numbers [25]. Rourke and Conway (1997) said that remedial education should put more emphasis on interventions that are right for the underlying neuropsychological

perceptual and visuospatial or verbal and auditory perceptual problems[18]. A pocket calculator may be helpful for children who have trouble solving math problems because they can't remember all the numbers they need to know [8]. Emersonet al. (2014), proposed some interesting games and activities to develop general concepts on number sense, place value, multiplication, division, and other general mathematical concepts among dyscalculia children[5].Henderson (2013), proposed some intervention strategies and teaching techniques to develop early numeracy skills, and calculations among dyscalculia children[11].

Connection Between Dyscalculia and Dyslexia

Dyscalculia is just as common as dyslexia, but teachers, parents, school officials, and the government don't pay as much attention to it. According to Diwan (2008), children with dyscalculia are more likely to have it than children with dyslexia or word blindness. But, unlike dyslexia, not much is known about how common it is, what causes it, or how to treat it [4]. According to Hannell (2005), between 20 and 60 percent of the students have dyslexia and dyscalculia. Deficits in language and memory can make learning mathematics and literacy skills more difficult [9]. Studies by Willburger et.al. (2008) show that the cognitive basis of dyslexia and dyscalculia are independent of one other in another investigation on naming speed in dyslexia and dyscalculia [27].

Normal Children and Dyscalculia

The study by Owen (2005) comparing mathematical problem-solving errors among 3rd children with learning disorders and students without disorders found that students with learning disorders made more misspellings in problem translation than in calculation[15]. When Hartmann and Ann (2008) compared students with math learning disabilities to students with low math achievement in solving word problems, they discovered that students with low math achievement had more computational errors but fewer translation errors than studentswith math learning disabilities who had misconceptions in the areas of analysing, reasoning, and abstract thinking [10]. Mussolin et.al. (2010) discovered that, regardless of the number format, DD children showed a higher numerical distance effect than control children when comparing symbolic and non-symbolic numbers in children with and without dyscalculia [14].

Conclusion

Dyscalculia is a developmental disorder that needs help outside of what is taught in the classroom. The roles of special educators, medical experts, and the rapists are all part of the special support. Dyscalculia needs help to learn normally, and this can be done through early detection and specialized ways of teaching. Rehabilitation plans for these children need to be made based on how poor their problems are and what kind they are. For students to do well in school, they must learn how to do mental arithmetic, so there is a strong need for research in this area. Teaching a child with dyscalculia is a tough job, but teachers can feel more confident about doing it if they know how to help the child learn math better. Curriculum developers need to pay attention to the needs of these intellectually challenged individuals and support them by giving enough funds to

researchers to help them find ways to make matheasier to learn and to help as many schools as possible build the infrastructure needed for inclusive learning.

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Mathematics as a Tool for Astronomy: Measuring Cosmic Ages

Gourav Banerjee*

Introduction

Scientists present their evidence in the form of a measurement. People make many statements in everyday life. Some are qualitative and some are quantitative. In science, we try to deal only with statements that can be quantified. Otherwise, we would have no way to compare results. Scientists use measurements to arrive at precise and quantitative statements. Mathematics is the tool through which scientists study such topics like quantity. Regarded as the language of nature, mathematics has acted as the most essential weapon in almost every known field, including natural science, engineering, medicine, finance, and the social sciences.

Astronomy, the oldest of all sciences (perhaps after medicine), has its origins that stretch back to prehistoric periods. The subject of astronomy is the study of all kinds of matter and radiation beyond the Earth. It is an excellent vehicle for studying science because of the sheer diversity of the universe. Mathematics is and has been extensively used in studying astronomy right from the beginning. This is because astronomers need to handle very large and very small numbers for decoding the mysteries of the universe.

Earth's age: the quest begins

During any cloudless night, if you go away from the city in a place free from light pollution and if you look up towards the night sky, you will be mesmerized by a spectacular sight of thousands of stars dazzling in the sky. Naturally various questions will come to your mind like: Who created this Universe? How and why did it begin? What is the fate of the Universe? Who are we? How we came from? Are we alone in this Universe? Twinkling like diamonds, the beauty of the night sky has indeed captured the imagination of human minds since prehistoric times. Curious by nature, since our first awakening, Homo sapiens or humans have been asking these questions. This natural curiosity of humanity gave rise to science.

If one looks at the long march of human understanding, one will find that science begins with the idea that we can understand our role in the universe using logic and observation. The stars and planets were mysterious objects for most of human history. Yet ancient Greek philosophers were able to use reasoned arguments and simple geometry to estimate the sizes and distances of our neighbors in space. Similarly, the most important scientific step in understanding Earth's history was not one particular set of measurements, but rather the mental leap of realizing that Earth had a history that could be unraveled by scientific observations and measurements. This happened mostly in the 16th and 17th centuries with the Renaissance and then with a period of time called the Enlightenment. One example of how science works is humanity's long, step-by-step search to find the age of the Earth. By studying the Earth and the Moon, scientists have been able to piece together their linked histories.

In the Middle Ages, scholars thought they could calculate Earth's age by finding out how long humans had lived on Earth. They assumed that humanity had been around almost

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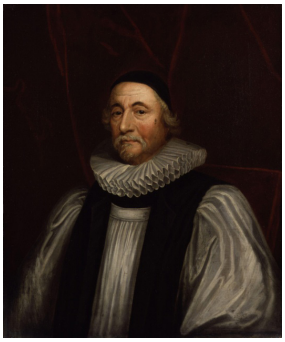
since Earth itself formed. Therefore, scholars analyzed ancient records, especially Biblical scriptures that listed the generations since Adam and Eve. They concluded that humanity, Earth, and the whole cosmos were only a few thousand years old. The most famous calculation was made by Irish Archbishop James Ussher in 1650. Ussher deduced that the cosmos formed on Sunday, October 23, in 4004 B.C., and that humanity was created on Friday, October 28 the same year. Even today, some people — especially fundamentalist religious groups — still believe that the Earth is only a few thousand years old, based on this method of reasoning.



James Ussher, the person who first deduced that Earth was formed in 4004 B.C (credit: www.wikipedia.org)

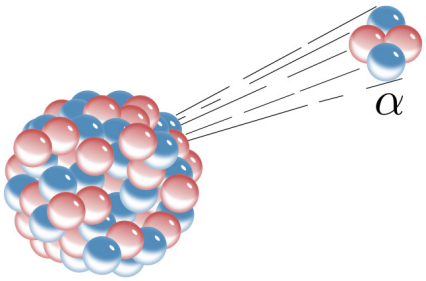
Then, gradually evidence mounted during the 19th century that the Earth was far older than previously believed. Presently we know with reasonable confidence that our planet and its only satellite, the Moon, are a double system that formed around 460 crore (i.e. 4.6 billion) years ago. The Moon probably originated during a gigantic collision in the late stages of planetary formation, after the Earth's iron core formed. But, which weapon helps us to probe the age of the Earth and Moon? How can we tell the age of so confidently? Here comes the role of mathematics. Yes, mathematics equipped with brilliant minds of scientists became the invincible tool which ultimately revealed the age of our planet and its neighbour. Without mentioning the detail history of how this feat was achieved, I shall now discuss the mathematical technique which we use to measure the ages of our Earth, any rock, fossil or even any old architechtural artifacts.

Radioactive decay process

The age of the Earth-Moon system and the chronology of the Earth's history are measured using the technique of radioactive decay. This well-understood physical process also provides the energy that drives most of the Earth's geological evolution. In the late 1800s, the crucial discovery of radioactivity made possible even more accurate estimates of the Earth's age. The process is worth discussing in detail since it allows us to date not only Earth but also rock samples from other worlds, such as lunar rocks and meteorites.

The discovery of radioactivity happened by accident. In 1896 French physicist Antoine Henri Becquerel left some photographic plates in a drawer with some uranium-bearing minerals. Later he opened the drawer and found the plates fogged. Being a good scientist, he did not dismiss the event but investigated further. He found that the uranium emitted "rays," which, like X-rays (discovered the previous year), could pass through cardboard.

The new "rays" turned out to be not electromagnetic radiation, like ultraviolet light or X-rays, but rather energetic particles emitted by unstable atoms.



Alpha decay is one example type of radioactive decay, in which an atomic nucleus emits an alpha particle, and thereby transforms (or 'decays') into an atom with a mass number 4 less and atomic number 2 less (credit: www.wikipedia.org)

Here is how radioactivity works. A radioactive atom is an unstable atom that spontaneously changes (usually into a more stable form) by emitting one or more particles from its nucleus. The original atom thus becomes either a new element (change in the number of protons in the nucleus) or a new form of the same element, called an isotope (change in the number of neutrons in the nucleus). The original atom is called the parent isotope and the new atom is called the daughter isotope.

The time required for half of the atoms of any original radioactive parent isotope to decay into daughter isotopes is called the half-life of the radioactive element. If a billion atoms of a parent isotope were present in a certain mineral specimen, a half billion would be left after one half-life, a quarter billion after the second half-life, and so on. Sometimes the result of the decay is another radioactive element, so the decay continues. But in every case, the final product of the decay (or chain of decays) is a stable element. For example, carbon-14 decays to nitrogen-14 with a half-life of only 5,570 years. Likewise, uranium-238 decays (in a series of steps) to lead-206 with a half-life of 4.5 billion (1 billion = 100 crore) years. Radioactivity is a random process. This means that the exact time when an individual atom decays is impossible to determine. Yet the average time for half of a very large number of atoms to decay is well determined.

Radioactive Half-Life

Using the general principles of radioactive decay, scientists have developed a simple equation to calculate a rock's age. Let us examine how a scientist might use these principles to develop such an equation. Our goal is to give the age of a rock sample in terms of the number of atoms that have decayed. Suppose a radioactive isotope has a half-life of 1 million (1 million = 10 lakhs) years. A certain number of these atoms are trapped in a crystal as molten lava cools to form a rock. After 1 million years, half of them would be left. After 2 million years, half of that amount would be left. This would be ½ × ½ = ¼ of the original number. How many would be left after 3 half-lives, or 3 million years? It would half that number again, or ½ × ½ × ½ = [!. From the progression of half-lives,

it is easy to see how to convert from this specific description to a formula for the general case. Let's define the fraction of the atoms that are left as F. Suppose that we wait N half-lives, and ask how many atoms would be left. Based on the paragraph above, we see that the answer would be:

$$\mathbf{F = (½)\ N}$$

To make the same equation even more useful if you have a calculator, we can take the logarithm of both sides (base 10 logarithm), which gives:

$$\mathbf{\log F = N \log (½) = -0.301\ N}$$

We can check this result. From the discussion above, we know that when N = 3 halflives, the fraction of atoms left is [!. Substituting N = 3, we get log F = -0.903. A calculator confirms F = 0.125, or [!.

Suppose we want to determine the age of a rock crystal and its potassium atoms. A particular radioactive form of potassium decays with a half-life of 1.25 billion years (known to 3 significant digits), yielding a certain form of argon atoms. Suppose we measure the argon and potassium in the rock crystal, and we find that 58% of the radioactive potassium has already decayed into argon, while 42% of the original radioactive potassium atoms are left in the crystal. How old is the rock? Our measurement has told us that F is 0.42, and so our equation gives -0.376 = -0.301 N. Thus, N = 1.25 half-lives. That would mean that the rock is 1.62 billion years old.

Ages of the Earth and Moon

Understanding the age of the Earth was more difficult than early scientists imagined. The answer couldn't be found by counting generations of humans. The answer couldn't be found by measuring sedimentary layers. It wasn't until radioactive decay processes started to be understood in the 1890s that people could begin to get an accurate measure of the age of both the Earth and the Moon.

How did scientists pin down the ages of the Earth and the Moon? Using radioactive dating of rocks, we can measure the time since the rock was last melted. When rocks form from molten material, parent (and forming daughter) isotopes get locked into a single location. Prior to the rock's formation, the products of radioactive decays could escape (for instance in the mixing of liquid lava). Once the rock solidifies, decays happen in situ. By knowing the rate of decay processes, and measuring the ratio of parent and daughter isotopes, it's possible to place constraints on the age of a rock.

Armed with the technique of radioactive dating, geologists set out to find the oldest rocks on Earth. These oldest rocks are very rare because, after billions of years, most have been destroyed by the Earth's active geology. The oldest regions of unaltered crust are in Greenland, Western Australia, and South Africa, and they are about 3.9 to 4.0 billion years old. In one remarkable discovery from the Jack Hills region of Western Australia, a small zircon crystal was found with a radiometric age of 4.404 billion years. Think of it — a mineral smaller than the head of a nail that tells us about billions of years of history! We have to add to this age an uncertain number representing the time the Earth was molten and the surface was being heavily bombarded by Solar System debris. An upper bound

to the age of the Earth comes from the most primitive meteorite materials, which are inclusions rich in calcium and aluminum. This measured age of the Solar System is 4.567 billion years. The best estimate for the total age of the Earth is 4.54 billion years with an uncertainty of only 1%.

Lunar Olivine Basalt 15555 sample collected from the Moon by the Apollo 15 mission. It was formed around 330 crore (or 3.3 billion years ago. It is on display in the National Museum of Natural History of USA

(credit: https://commons.wikimedia.org/wiki/ File: Lunar_Olivine_Basalt_ 15555_ from_Apollo_15_in_National_Museum_of_Natural_History.jpg)

To verify this number, scientists also wanted to date lunar rocks to see if they were comparable in age to the Earth. Each Apollo mission brought back a precious cargo of lunar rocks, soil samples, and drill cores. Together with some smaller samples brought back by earlier, unmanned Soviet probes, scientists gathered several hundred kilograms of rocks from nine different lunar sites. Lunar scientists eagerly tested these samples and began to unravel the history of the Moon. Many of the rocks were in the range of 3 to 4 billion years old, with a few chips dating back to 4.4 or 4.5 billion years ago. Apparently, most rocks from the first few hundred million years of the Moon's history were pulverized and destroyed by the intense bombardment that occurred at that time, which is why the earliest rocks and chips are rare. Scientists add about 100 million years to this age for the time it took the molten Moon to solidify. The best estimate of the age of the Moon from radiometric dating is 4.527 billion years, slightly younger than the Earth.

Absolute ages from radiometric dating were used to calibrate the relative ages of different parts of the lunar surface from crater-counting methods. Radioactive dating confirmed what the lunar cratering record indicated: the smooth dark plains, the maria, are considerably younger than the heavily cratered highlands. Once relative ages from the cratering record were calibrated, absolute ages could also be found for other planetary surfaces, such as Mercury and Mars by assuming cratering rates were consistent throughout the inner solar system.

Age of Solar System

The methods of radiometric dating have been refined over sixty years, and dozens of different radioactive species can be used for this research. When we apply the radioactive dating technique to all sorts of Solar System samples — terrestrial rocks, lunar rocks, and meteorites from deep space — they all yield an age of approximately 4.6 billion years. The technique is based on the well-understood physics of radioactivity, which can be tested in the laboratory, blended with in-depth knowledge of mathematics. Scientists consider this proof of the age of the Earth, and it also led to the chronology of the entire Solar System. Consider this vast number for a moment. Compared to this, all the generations of humanity are only the blink of a cosmic eye. We've been around for less than a tenth of one percent of the history of our planet!

The story continues

The story does not end here. It literally cannot. This is because the curiosity of human beings is unquenchable. Naturally any curious mind will ask another question, what is the age of the universe? Fortunately, the power of human intellect combined with the supreme efficiency of mathematics and physics have provided an answer to this almost impossible to solve question also. We now know with reasonable confidence that our universe is around 13.8 billion (i.e. 1380 crore since 1 billion = 100 crore) years old. But, how do we measure the age of the universe? To know the answer, one has to study 'cosmology'. This might be one of the most thrilling detective stories of all time.

Cosmology is the study of the size, structure and evolution of the universe — in other words, the "geography" of the universe as a single system. In cosmology, our subject is the whole universe, defined as all matter and energy in existence anywhere, observable or not.

Birth of cosmology

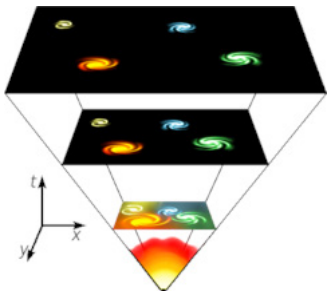
One of the most fundamental questions of humanity is the question of origins: Where did we come from? This question runs deeper than the origins of any one person, and delves into questions of where did life, our planet, and the entire universe come from. Many cultures have avoided the notion of an origin to the universe by placing their creation stories in a cycle. Buddhist and Hindu origins stories measure the birth, death, and rebirth of the universe in units of 4 trillion years, which is a day in the life of Brahma. At night, all matter is absorbed into the spirit of the sleeping Brahma. At dawn, when Brahma awakes, matter reappears and the cycle continues. The Greek Stoics saw the universe as being created from fire, only to be destroyed by fire, and so on. Cosmological cycles are also found in the cultures of the Maya and the Aztec.

The story of modern cosmology began with Albert Einstein and Edwin Hubble. From Einstein's theories, the shape of space is described as being warped by gravity. From the observations of Hubble, we learned the universe is expanding. Together these two men described the evolving shape of space and they opened up a new field of science called cosmology.

The Big Bang model

The modern theory of the origin of the universe starts with the idea of expanding

space. Georges Lemaître was a priest and a mathematician, an unassuming man who had beaten the giants of physics to the punch in deducing that the universe could be expanding. In 1929, Lemaître was the first to hypothesize a revolutionary idea: at one time, the universe might have been as small as an atomic nucleus. He proposed that the universe derived from a cosmic singularity, "a day without a yesterday" when the universe was infinitely small and infinitely curved, and all matter and energy were concentrated in a single point. Many astrophysicists found the idea bizarre and distasteful and English theorist Fred Hoyle disparaged the idea with the name "big bang." The label stuck, and scientists continue to call the description of the creation of the universe the Big Bang model.



According to the Big Bang model, the universe expanded from an extremely dense and hot state and continues to expand today (credit: www.wikipedia.org)

The Big Bang model forces us to consider the idea of cosmic evolution. The universal recession of galaxies implies that the universe is evolving, and it has not always been in the same state. Our observations of nearby galaxies represent a single frame in a movie that has been playing for billions of years. We can wind the tape backwards, seeing snapshots of a progressively older universe, by observing sets of galaxies at progressively larger distances, and thus at progressively larger look back times. (Since light takes time to travel, we see objects far away as they looked when the light was released rather than as they are now.)

As we look back into the early universe, we can see the galaxies move closer together. Observationally, we can't see all the way back to the first moments, but we can see times when galaxies were just forming, and the large scale structures — the cosmic walls and voids — were just starting to form. If we could look further back, we'd see that as the universe gets smaller; its volume contracts until all the masses are smashed together in a tiny universe. The mass that formed galaxies and stars breaks down into a seething hot gas, as the universe tends toward a state of infinite temperature and density. Thinking of the Big Bang as an explosion is tempting, but it is also misleading. In an explosion on Earth, debris flies through space. In the Big Bang, the initial singularity contains all space and matter. Time itself begins with the big bang. The evolution of the universe is the unfolding of time and space from a condition of incredible heat and density to cold and enormous state billions of years later. This bang had no center, it had no past and it simply was the sudden beginning of everything in an expanding geometry that may or may not be finite.

The scientific story of creation says that everything — you and the Earth and the Sun

and the Milky Way and all the billions of galaxies — emerged from a tiny dense dot of energy and matter that unfolded into the universe we see now. It sounds as fantastic as any of the creation myths of older cultures. How do we know that the big bang actually occurred? There are three primary pieces of evidence:

• Galaxies are taking part in a universal expansion as indicated by the linear relation between distance and redshift (the Hubble relation).

• The abundance of the lightest elements can be explained by fusion in the universe when it was young and dense and hot (cosmic nucleosynthesis).

• Space is filled with the radiation from the early hot phase, now diluted and reduced in energy to the level of microwaves (the cosmic microwave background).

Describing each of these concepts is beyond the scope of this article. But the Hubble relation mentioned in the first point bears huge importance in measure the age of the universe. Hence, I shall take the opportunity to discuss in brief about the Hubble relation.

Hubble relation or Hubble's law

The Hubble relation, popularly known as the Hubble's law or Hubble–Lemaître law is considered the first observational basis for the expansion of the universe and today serves as one of the pieces of evidence most often cited in support of the Big Bang model. Although widely attributed to Edwin Hubble, the notion of the universe expanding at a calculable rate was first derived from the general relativity equations in 1922 by Alexander Friedmann. Friedmann published a set of equations, now known as the Friedmann equations, showing that the universe might expand, and presenting the expansion speed if this was the case. Then Georges Lemaître, in a 1927 article, independently derived that the universe might be expanding, observed the proportionality between recessional velocity of and distance to distant bodies, and suggested an estimated value of the proportionality constant, which when corrected by Hubble became known as the Hubble constant. Then in 1929, Edwin Hubble confirmed the existence of cosmic expansion, and determined a more accurate value for the constant that now bears his name. Hubble inferred the recession velocity of the objects from their redshifts, many of which were earlier measured and related to velocity by Vesto Slipher in 1917.

Georges Lemaître**, Edwin Hubble and Fred Hoyle (credit:** www.wikipedia.org)

The law is often expressed by the equation:

$$v = H_0D, \text{ where} \quad (1)$$

- v is the recessional velocity, typically expressed in km/s.
- H_0 is Hubble's constant and corresponds to the value of H (often termed the Hubble parameter which is a value that is time dependent and which can be expressed in terms of the scale factor) in the Friedmann equations taken at the time of observation denoted by the subscript 0. This value is the same throughout the universe for a given comoving time.
- D is the proper distance (which can change over time, unlike the comoving distance, which is constant) from the galaxy to the observer, measured in mega parsecs (Mpc), in the 3-space defined by given cosmological time.

The reciprocal of H_0 is the Hubble time. This constant term H_0 help us to estimate the age of the universe. Let me now discuss in brief how it does so.

Age of the Universe

When Einstein, Lemaitre, and others solved the equations of general relativity, they were able to describe how the size of the universe has changed with time. Astronomers use the symbol R to represent the scale or size of the universe at any time. You can think of R as the size of the universe, but more accurately it represents the distance between any two well-separated places. The cosmological principle says that any two points are moving apart at the same rate. Thus, the entire history of the universe is described by the way that R varies with time. Since the universe is expanding, R has been continuously increasing for billions of years. Remember that R describes the expansion of space that carries galaxies apart — the galaxies themselves are not expanding. Galaxies are just markers of expanding space.

The Big Bang model even provides a prediction for the age of the universe. If we imagine the evolving universe as a movie, the birth of the universe is the time in the distant past when all matter and radiation was crushed in a state of infinite temperature and density. The scale factor, R, was zero. Space had not yet begun to unfold. There are three ways to constrain the age of the universe. But in many ways, looking at the expansion rate of the universe is one of the easiest ways to figure out the universe's age. This technique looks at the current expansion rate and then running it backwards to determine when today's universe must have been compressed down to a single point. Mathematically, the age of the universe is:

$$t_0 = 1 / H_0 f (\Omega_r \Omega_m \Omega_{\gamma}...) \quad (2)$$

Where t_0 is the current age, H_0 is the current expansion rate, and f is a correction that takes into consideration the mass, energy, and other characteristics of the universe. For this method to work, you need to not only know the current expansion rate but also how this rate has changed over time due to the effects of gravity and the effect of dark energy ($\dot{U}_{\ddot{e}}$), which is pushing the universe apart. The best published value for the expansion rate is H_0=70.6 ± 3.1 (km/sec) / Mpc. This means that every Megaparsec of space expands

roughly 70.6 km each second.

Based on stellar evolution models, we estimate the oldest globular star clusters are between 11 and 13 billion years old. Apart from these two techniques, the most trusted means of determining the age of the universe comes from very precise measurements of the cosmic microwave background (CMB) radiation. The CMB is a diffuse, uniform background of microwave radiation that comes from all directions in the sky. This CMB is the electromagnetic radiation which is the remnant from an early stage of the universe, also known as "relic radiation". First observed accidentally by the radio engineers Arno Penzias and Robert Wilson in 1965, CMB is a crucial piece of evidence that supports the big bang model for the origin of the universe.

Three space missions in particular have been instrumental in measuring this radiation in finer and finer detail and with better and better precision. NASA's Cosmic Background Explorer (COBE) took data from 1989 to 1996 and it was the first to detect tiny variations in the intensity of the radiation. Next came the Wilkinson Microwave Anisotropy Probe (WMAP), launched by NASA in 2001. WMAP took data for a decade and its very accurate measurements put constraints on a set of parameters of the big bang model. The current state-of-the-art experiment is ESA's Planck mission, launched in 2009. The level of accuracy of Planck data is tens of thousands of times greater than the discovery measurements of Penzias and Wilson. Based on the WMAP measurements and theoretical models, the universe is estimated to be 13.82 ± 0.12 billion years old. This is the most quoted age used by astronomers and is consistent with both other measurements within their errors. Remarkably, you can see that this value is very close to what has been estimated using the Hubble constant in equation 2.

Conclusion

Very early in the history of mankind, ignorance of nature and fear of the unknown naturally made men religious. So God played a central role in primitive societies. They tried to answer every question with the help of God. As time passed, with experience and intellect men could understand more and more of the natural phenomenon. Birth of science took place with the idea that we can understand our role in the universe using logic and observation. Today humans have come a long way. And mathematics has truly acted as the most essential weapon in almost every known field, including astronomy. By now any reader will understand how mathematics in simple form also has helped in the development of astronomy and human knowledge as a whole.

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Online Mathematics Teaching During COVID-19 Pandemic

Kamal Krishna De*

During the pandemic of COVID-19 all education particularly , face to face school education system(the offline education system) switched over to online education. Like education of Languages and Social SciencesubjectsMathematics and Science were also subjected to online education system. Whether the online education of Mathhematics attained the similar height as in Social Science and Languageeducation, had been a glaring topic for discussioneverywhere.

Before the pandemic

1. The authorities of many institutions didnot allow any phone with the school students.
2. The use of smart phone by the students would invite the ire of the school authority and the society at large.
3. (a) When the objectives of education were more focussed on market than on the individual development of the students and
   (b) When the engieering education began to relegate to background, students also began to turn face from mathematics and mathematical education at schools. Even a cursory look at higher secondary classes in our schools would present the fact that most of the students offer Mathematics as a 4th elective. This was unbelivale three decades ago.
4. The standard of mathematics teaching at remotes area of the countryis awesome. At the advent of lockdown following Covid Pandemic,countries likePhillipins, Brazil, Latin America,Indiaand some other developing nations felt helplessness in teaching mathematics online as the major approach. Mathematics is symbolic, digrammatic and less verbal. Its teaching requires a very conscious, accurate and logical interaction among teacher and students. Clear,prominent,consistent projections or presentation on (black/white)classroom board and meticulous observation by the students accelerate understandingin mathematics.In normal classrooms of the schools teacher becomes tired in teaching mathematics due to some lacune on the parts of teacher, students and schools specially in Govt controlled schools.

   When teachers of the schools under the control of Govt are at a loss how to teach the students under the unexpected situation, the teachers of private institutions had started slowly but firmly teaching mathematics online in the schools following the practices of developed countries all over the world. We were simply left to see the gradual regression of Govt controlled schools in this arena.

   In spite of social restrictions(as noted above), by and large our student communities were conversant in games,correpondence, calculation,music , cinema etc. throughfourth generation (4G) smart phones but they were not so promt in utilizing the phone for teaching leaning purpose.Teachers were mostly satisfied with theircomperatively backdated old second generation model (2G) mobile phones. It was a tremendousjerk for both the groups to take part in online teaching learning through smart phones (or laptops) during the COVID-19 Pandemic . It took nearly one year to provide laptops or smart phones to all concerned and to trainall the teachers

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in how to use accessories for online teaching-learning of mathematics. In langage and and social science, online teaching achieved desired success but in mathematics and science, online teaching cut a sorry figure.

The interaction in online teaching in mathematics has proved to be highly insipid to a large section of students.Mathematical knowledge is systematic, continuous and highly intelinked.Any pause or break anywhere in the process of learning due to lack of attention, sincerityand regularity on the part of a student can dismantle forever the mathematical sructure formed or developed within the student. This is to be kept in mind by the online mathematics teacher. In the face to face offline teaching, any weakness on the part of a teacher or any limitation in mathematics teaching could be sometimes replenished (or absorbed) by intensive interaction between teacher and the students. But this scope is not always available in online teaching. Psychologically an online mathematics class spanning over more than two or three periods heavily taxes the students'receptive mechanism. So the duration of an online mathematics class should be set optimally. The branches of mathematics likearithmetic, geometry, mensuration, trigonometry could make a headway in online teaching. Even online teaching sometimes seemed to be effective in presentation of introductory lessons on algebra and some algebraic formulae. But as a whole,the online teaching of algebra remained shadowy as reported by some participating school students.

Some aged and senior mathematics teachers thought that online education would resemble offline or face to face mode of teaching mathematics. But their speculations were shattered by the stroke of reality. Sometimes the teachers belonging to this group did not go for training in online teaching. It may not be out of place to mention here that some of the aged teachers are very successful in offline mode but in onlinemode they could not maintain the same record.

The science of teaching almost remaining same, the techniques(or art) of teaching have to be changedto suit the online mode to make the lessons more interesting and enjoyable as in offline. In online mathematics teaching if a student develops apathy at some point it will be very difficult to remove thatin future by the online mode of teaching.

Discussion on how to make mathematics lesson interesting in online mode of teaching:

■ For introducing a new concept/chapter/exercise some references might be there to the real life situations.

■ The students can generally feel the presence of mathematics in differnt situations of life but they can't gauge how mathematics work.They have the knowledge of part and whole, greater/less, use of number tocount different articles,distribution of of different number of items.This idea can be usefully exploited in teaching ratio, division, measurements, LCM, HCF etc.

■ Ratio (proportion of different spices in preparing a food item)

■ LCM & HCF (distribution of food packets,chokolates etc., & setting of tiles or bricks along different sides of aroom)

- Percentage & Profit and Loss(cutting or joining paper,growth of a plant)
- Measurement by guessing (rate of walking, distance covered)

Variation : The concepts of variation are generally given by the teacher with the help of Charles' law, straight line or by land cultivation etc (for direct variation) and Boyle's Law law,asymtote(inverse variation). But they may be presented in a recreative manner)

Direct Variation: Out of two variables when one increases, the other also increases; the phenomenon may be demonstrated by sizes of letters. When out of two letters size of one letter increases, the size of the other also increases.

Decrease or increase of both the letters shownwith the help of adjacent angles

Fig.1: Direct variation

Inverse Variation: Out of two variables when one increases, the other decreases and vice versa; the phenomenon couldalso be demonstrated by sizes of letters. When out of two letters the size of one letter increases the other decreases.

Increase of one letter with the decrease of other shown with the help of alternate anglescontained in letterZ

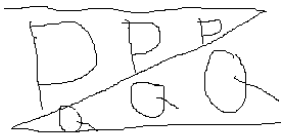

Fig 2: Inverse Variation

For smooth progress of online teaching in mathematics we can maintain the following schedule:

- Class routine should be circulated well in advance to the students.A whatsapp group may be formed to ease the process. The problems may be given to the students beforehand so that they may be physically and mentally prepared to join the the online program with those problems in mind.If the students are provided with PHOTOMATH App, they may take the help of it for problem solving when get chance to use it .
- The syllabus of mathematics may be scanned at every stage of learning toretain only the most essential parts which have contributions to the development of the students themselves and also also have impact on the daily life of them.So far it is known, WB Board of Secondary Education took such venture for the benefit of the students during the COVID-19 Pandemic.
- Inorder to tackle the students backward in mathematics, the students may be given lesson using initiallythe Deductive method of teaching. In comparison to offline mode a little more time should be allotted for teaching mathematics online.

NIT Durgapurarranged online mathematics education for the school students from the

community halls/school buildings in the neighbouring rural areas,using computers and large projectors during lockdown period.

A study in South Africa has shown that use of smart phones have done a miracle in learning mathematics online by the marginalized students of that country. The method used for the learning is called as ERT(Emergency Remote Teaching)

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****The Bengali version of this article was read by the author in the Annual Conference (online) of CPSM during 2021.**

Pivotal Role of Mathematics in Astronomy and Need of Awarness of Astronomy in School Level Education

Ashmita Trivedy

Twinkle twinkle little star
How I wonder what your ar.
Up above the world so high
Like a diamond in the sky.

Yes!! These were the first lines of out childhood English rhymes, we learnt when we went to the primary or nursery schools. So we can very well assume that when these lines were written.... human mind had a quest about the 'Night Sky". The existence of star studded sparkling and twinkling stars raise and developed numerous question in human mind Like how? Why? And when? These stars were positioned? Why are they not falling from sky? Do the move/ if yes, then form where to where? Where do the go during daytime? Importance of sun.

So, these numerous questions had their answers in the subject called ASTRONMY OR JOTIRBIGYAN (as named in India). The subject is concerned about "TE STUDY OF OBJECTS OUTSIDE THE EARTH'S ATMOSPHERE.

In ancient civilizations like Egypt, India, China, Babylon, Mesopotamia, Mexico and Greece we find lot of study of astronomy and its mathematical calculations through inscriptions, seals, stone carvings on monuments. The most relevant purpose was for their 'Navigation' and 'Agriculture'. When people move from one place to another both on land and sea. They followed the movement of stars in night sky and calculated their position. Transportation was very much dependent on calculations when people wanted to explore new lands and do trade.

Agriculture was dependent on astronomy because on their mathematical calculations and observations of stars they found the different sesonal rainfall, the period of draught, the tidal waves and their relationship with moon, sun rise and sunset. The climatic changes and its affect on agriculture.

As we see different discoveries from ancient times to modern times astronomy has evolved as a beautiful dynamic subject, all based on mathematical calculations and observations. These calculations are developing and evolving with more new approaches in recent times.... so we can assume Mathematics play a pivotal role in Astronomy.

As Tegmarks view—Everything in universe—humens included is a part of mathematicsl structure. All matter is made up of particles, which have properties such charge and spin, but they are purely mathemarical.

Different discoveries through ages from ancient to modern times have fascinating stories of different astronomers, laws and theories were developed. Galilio Galiei in 1610 used a newly developed telescope to see the sky and observed different movements of stars and planets, wrote many theories and recorded observations, later was again verified. Human rac echronoogically got impression of stars and plantes. Much laer Nicholas Copernicus who was considered as father of modern Astronomy with his mathematical calculations derived that "earth and other plants are moving around sun" Thus earth and other planets

have rvolving motion.

In ancient India too there was astronomers like Aryabhatta and Varahamihira. From 3rd century astronomy was developing in India and by 6th eentury Aryabhatta formulated the calculation of the area of a triangle, which led the origin of Trigonometry, but Greek knowledge influenced Indian Astronmy.

The ancient Indians made three distinct contributions in theoretical astronomy based on mathematical calculations. They are, (a) Notation system—found in the inscription of king Ashoka. (b) The decimal system and (c) The use of Zero..... the Indians considered zero as a separate number.

In India there are so many astronomical instruments build durign the different reign of kings or Rjas, the most famous are the Sun Dials like Jnatar Mantar in Jaipur, Delhi and Varanasi. In Jaipur all the 27 instruments are still in working condition.

Over the years astronomy got huge contributions of discoveries and inventions. And in more recent times Albert Einstein's theory of relativity, gravitation and his understanding of molecules have defined new approaches in field of astronomy astrophysics.

Thus, we can say that through different period astronomy became and ever veolving subject with different facets and dimensions. It is also considered as the oldest science.

To quench the thirst for the unknown universe this subject has an utmost need to be included in school level education in secondary level. But to our great dismay we do not find this in the curriculum in early years of school level. It can be imbedded in different relevant subjects like as solar system in geography, earth science, astrobiology in life science, astrophysics in physics, astrogeology in geloogy and chemistry etc. It is always thought to be a far reachable subject and can only be learned if the child has in-depth knowledge of only maths and physics. Thus, it only reachable after higher studies, but we need to rise the curiosity and interest int he child's mind from school level which make the child aware of the new development in this field.

Astronomy can be a fun-loving subject for school children through hands on activity, sky watching with and also without telescope, watching sun and moon eclipse with proper measure, making school projects on astronmy, keeping records of observations. These can make the child engross in the subject ant satisfy their curiosity. Which in later years can become their carrier for bright future which has a global opportunity. Astronomy has become a global theme and study where huge number amateur astronomers of amateur asronomical organisations provide and contribute recorded data and results of observations to the professional organisations and scientist to help them to complete their projects and discoveries.

Such organisations for public contributions are zeeniverse, Citizen science, AAVSO and many more.

So we conclude that there is always a new hope for awareness in astronomy which shall motivates school children thus create a demand to include this subject in parts in the school curriculum of secondary level and make this vast subject more approachable, to the students who can quench the thirst of astronomy, to solve the mysteries of the UNKNOWN UNIVERSE FROM WHERE WE HAVE COME.

Solution of Equations by Vedic Method

Dr. Partha Karmakar*

There are several methods to solve algebraic equations. With the held of Vedic Mathematics solution of such particular algebraic equation can be done within a little time.

Type 1: **If the sum of numerators and the sum of denominators be the same, then the sum = zero.**

$$\frac{2x+3}{5x+6}=\frac{6x+7}{3x+4}$$

Here, we see that

$N_1 + N_2 = 2x + 3 + 6x + 7 = 8x + 10$

$D_1 + D_2 = 5x + 6 + 3x + 4 = 8x + 10$

Hence, the solution of the equation is obtained by

$8x + 10 = 0 \Rightarrow 8x = -10 \Rightarrow x = \frac{-10}{8} = \frac{-5}{4}$

N.B.: If there be a numerical factor in the algebraic total, that factor should be removed.

Let us discuss it with an example: $\frac{2x+3}{x+1}=\frac{6x+7}{3x+4}$

Here, $N_1 + N_2 = 2x + 3 + 6x + 7 = 8x + 10 = 2(4x + 5)$

$D_1 + D_2 = x + 1 + 3x + 4 = 4x + 5$

We see thus, $N_1 + N_2 = 2(4x + 5) = 2(D_1 + D_2)$

We can remove the numerical factor 2

Hence, solution in given by, $4x + 5 = 0 \Rightarrow x = \frac{-5}{4}$ **Ans.**

Type 2: **If the sum total of the denominators on the left hand side and right hand side are same and if the othe elements are equal, then the total is zero.**

Example: $\frac{1}{x+7}+\frac{1}{x+9}=\frac{1}{x+6}+\frac{1}{x+10}$

Here, $x + 7 + x + 9 = 2x + 16$ and $x + 6 + x + 10 = 2x + 16$

i.e., $D_1 + D_2 = D_3 + D_4$

Both total = $2x + 16$

Hence solution of this type of equation is obtained by $2x + 16 = 0$

Type 3: Quadratic Equation: Form $\frac{N_1}{D_1}=\frac{N_2}{D_2}$

If $N_1 + N_2 = N_3 + N_4$

Then one root is zero *i.e.*, $x = 0$

Also another root will be calculated using, $D_1 + D_2 = D_3 + D_4$

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Example: $\frac{4}{9x+4}+\frac{5}{9x+5}=\frac{1}{9x+1}+\frac{8}{9x+8}$

Here, $N_1 + N_2 = 4 + 5 = 9$ or, $N_3 + N_4 = 1 + 8 = 9$

$\therefore N_1 + N_2 = N_3 + N_4$

Hence, one root is $x = 0$

Now, $D_1 + D_2 = 9x + 4 + 4x + 5 = 18x + 9$,

$D_3 + D_4 = 9x + 1 + 4x + 8 = 18x + 9$

Other root is obtained by

$D_1 + D_2 = D_3 + D_4 = 0$ *i.e.*, $18x + 9 = 0 \Rightarrow 18x = -9 \Rightarrow x = \frac{-9}{18}$ **Ans.**

Type 4: Form $x+\frac{1}{x}=a+\frac{1}{a}$

Then the roots of this type of quadratic equation will be $x = a, \frac{1}{a}$

Example : (1) $x+\frac{1}{x}=\frac{17}{4}$ *i.e.*, $x+\frac{1}{x}=4\frac{1}{4}=4+\frac{1}{4}$

Hence, the solution are $x = 4, \frac{1}{4}$ **Ans.**

(2) $\frac{x}{x+1}+\frac{x+1}{x}=\frac{82}{9}$ *i.e.*, $\frac{x}{x+1}+\frac{x+1}{x}=9\frac{1}{9}=9+\frac{1}{9}$

Hence, roots are obtained by,

or, $\frac{x}{x+1}=\frac{-9}{8}$

or, $9x = -8x - 8$

or, $17x = -8$

$\therefore x = \frac{-8}{17}$

or, $9x - x = 1$

or, $\frac{x}{x+1} = \frac{1}{9}$

or, $9x = x + 1$

or, $8x = 1$

or, $x = \frac{1}{8}$

$\therefore$ The roots are $x = \frac{-9}{8}, \frac{1}{8}$ **Ans.**

Similarly we can solve:

(i) $\frac{x+1}{x+2}+\frac{x+2}{x+1}=\frac{37}{6}$ $\left[\textbf{Ans. } x=\frac{-11}{5}, \frac{-4}{5}\right]$ (ii) $\frac{x+4}{x-4}+\frac{x-4}{x+4}=\frac{10}{3}$ [**Ans.** $x = 8, -8$]

Only a few types are shown here. There are several other types of equations which can be solved using Vedic technique which may be discussed. later.

Mathematics Laboratory—An Alternative Method of Instruction

Subrata Kumar Biswas*

What is Mathematics Education?

- Mathematics education has traditionally emphasized the worst face of mathematics. Students are taught many mathematical facts, but they are rarely taught what the **mathematics process** is and how to employ it
- The mathematics process is both a creative and explorative process and is the most important face of mathematics.
- Every student of mathematics needs to learn the mathematics process.
- Mathematics is an indispensable part of education.
- Its practical utility is not understood by many. This has happened because of sheer ignorance of educators and learners of mathematics. This ignorance has led to scepticism or phobia of mathematics.

Recommendations under NPE(1986) and NCF(2005)

- Mathematics, according to NEP 1986, should be visualized as the vehicle to train a child to think, reason, analyze, and articulate logically.
- National Curriculum Framework 2005 **(NCF 2005)** also called for Mathematization of the child's mind.
- Mathematics should be treated as a concomitant to any subject involving analysis and reasoning.
- The teaching and learning of Mathematics should enable the child to consolidate the mathematical knowledge and skills acquired at Primary stage.
- Assimilated knowledge of mathematics should enable the child to solve the real-life problems by developing abilities to analyze, to articulate, to reason and to see inter-relationship involved, to think and reason.
- To develop the ability to articulate logically.
- To develop necessary & sufficient skills to work with modern technological devices such as calculators, computers etc.
- To develop appreciation for mathematics as a problem – solving tool in various fields, for its beautiful structures and patterns etc.
- So, to make the Teaching learning compatible to the above tenets, an alternate method of instruction or teaching method has to be evolved so that, teaching process of Mathematics at school level becomes effective and meaningful

Objectives of Mathematics Laboratory

- To provide readily accessible rich manipulative materials to emphasis on "learning by doing".
- To develop an attitude of enquiry.
- Remove the weakness of present day mathematics education.
- To develop much needed confidence in students.

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- To generate interest in the subject.
- To make the students divergent thinkers.
- To make the children to look for pattern and ask questions.

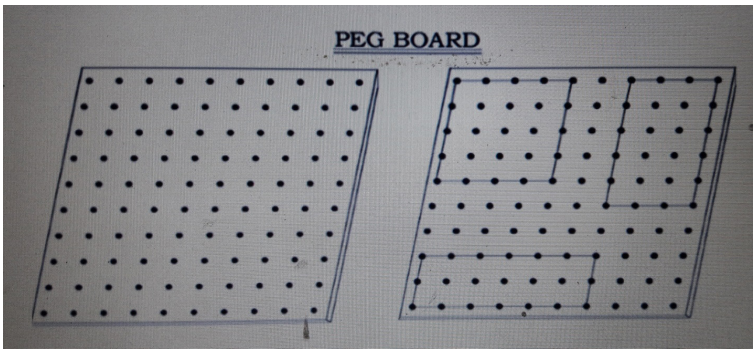
Need for a Mathematics Laboratory

- Situation through self exploration and discovery. He provides a solution based on his experience, needs and interest. Some of the ways in which a mathematics laboratory can contribute to the learning of the subject are.
- It provides an opportunity to understand and internalize the basic mathematical concepts through concrete objects and situations.
- It enables the students to verify or discover several geometrical properties and facts using models or by Mathematics laboratory is activity centred and a child is placed in problem solving paper cutting and folding technique.
- It helps the students to build interest and confidence in learning the subject
- The laboratory provides opportunity to exhibit the relatedness of mathematical concepts with everyday life.
- It provides greater scope for individual participation in the process of learning and becoming autonomous learners.
- It provides scope for greater involvement of both the mind and the hand which facilitate cognition.
- The laboratory allows and encourages students to think, discuss with each other and the teacher and assimilates the concepts in a more effective manner.
- It enables the teacher to demonstrate, explain and reinforce abstract mathematical ideas by using concrete objects, models, charts, graphs, pictures, posters etc.

Some Activities through Model/ Materials:

Area and Perimeter

Materials required: Square 10"× 10" board of acrylic/plywood/hardboard/chopped sole rubber, match sticks or short nails, string. Many high school students find it difficult to master the concept of perimeter even though it is a simple concept. Further, its relation to area, which is the other measure associated with a bounded plane figure, is not clear. Some students even have difficulty with the concept of area. This activity clarifies the notion of perimeter and helps students discover its distances from the concept of area. Drill holes on the square board at 1 inch intervals forming a square grid. The holes need to be just large enough to easily fit pegs which are about 2 mm in diameter. If the board is made of transparent acrylic sheet, it can also be used on the OHP. Another advantage with an acrylic sheet is that, it can be placed on tip of a graph paper, which can be used to estimate areas. (Materials such as thermocol, or chappal sole rubber can also be used if they are available easily.) Break the match sticks in half and scrape off the phosphorous for using them as pegs on the board. Alternatively short nails with small heads can be used. In all about a dozen pegs may be required.



The following activities can be performed with the pegboard:

- Take a length of string about a foot long and knot its two ends together to form a loop. Give the loop to students along with four pegs. What are the different kinds of figures which they can make? Ask them to name the figures. Ask them what remains the same as these figures change. Introduce the concept of perimeter as the total length of the boundary.
- Take a total string length of 20 inches which is knotted into a loop and four pegs. Ask the children to make rectangles of different sizes. Which rectangle has the largest area?
- With the same loop of string, ask the students to form figures with 3 pegs, with 5 pegs, 6 pegs and so on. Can they estimate the area? Which figure has the largest area?
- Ask students to construct figures with a given area and measure the perimeters. Which figure has the largest perimeter?

Geoboard

Materials required: Square board 10”× 10”of wood or acrylic, nails or pegs, superglue if acrylic board is used. A geoboard is a very useful device for introducing children to important topics in school geometry. It consists of an array of nails or pegs which are placed at equal distances on a square acrylic board making up a grid. The baseboard is made from wood or acrylic. Drill holes at equal distances forming a square grid. It is convenient to place the pegs at gaps of 2 cm. Nails can also be used, but the heads must be small. Acrylic boards with holes drilled in them are good for use on an overhead projector. Graph paper can also be placed below an acrylic board which is useful in estimating areas. The pegs or nails can be glued onto the acrylic board. Polygonal figures can be made on the geoboard by stretching rubber bands across the pegs. A number of activities which are suitable for students of different classes can performed on the geoboard.

Some of the activities that can be performed are

- Study of kinds of angles.
- Areas and perimeter
- Property of similar figures: Take two rubber bands and form the triangles one within the other. Are they similar? Why or why not?

- On the geoboard it is very easy to find areas of figures by completing rectangles. Hence it is easy to verify theorems about areas. For example, with two rubber bands the theorem, "the area of a triangle on the same base and between the same parallels is half that of the parallelogram" can be verified.
- Another activity on the geoboard is to construct right angle triangles such that none of the sides of the triangle is aligned along a row or a column. How many such triangles can be constructed? What is the relation between the sides of these triangles?
- An interesting activity to do with the geoboard is to discover the Pick's theorem. The steps for this activity are described briefly: – Make different polygonal shapes on the geoboard. Can we determine the areas of these polygonal figures? There is a simple formula for obtaining the area of a polygonal figure on the geoboard. It is not difficult to derive this formula. Try to! – First begin with a simple figure that is easy—the unit square. What happens to the square as you deform one of its four sides, keeping its other three sides intact? Make a change in one of the sides of the square and describe the result by answering the questions below:
  (i) What is happening to the number of boundary points? Is it increasing or decreasing or remaining the same?
  (ii) What is happening to the number of points in the interior the polygon? Is it increasing? Are any of the boundary points becoming interior points?
  (iii) Prepare a table for each step as you change one side of the square. Similarly start with a rectangle and observe what happens. Can you arrive at a general rule for the area of a polygon?

The general rule for areas of polygonal figures is given by the formula

$$A = B/2 + I - 1.$$

A is the area of the figure, B the number of boundary points, and I the number of interior points.

Conic Sections

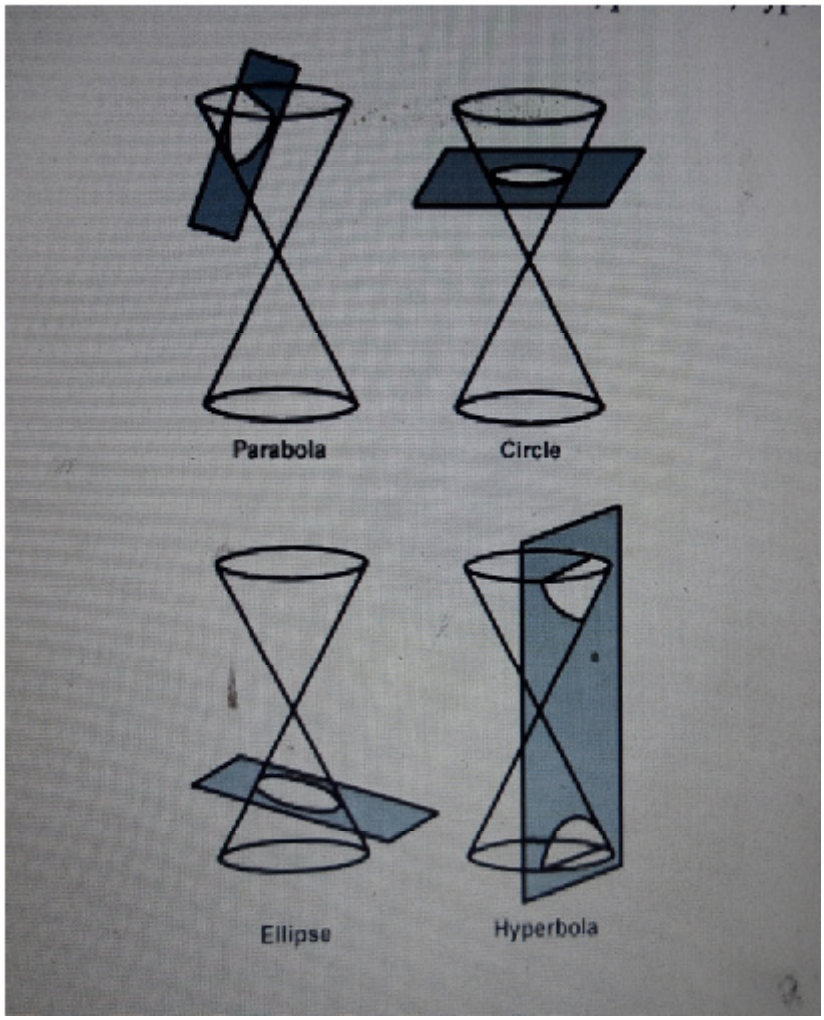
Materials required: Modeling clay, chart paper, tracing paper, thin wire, divider, plane paper, pen, wooden board, nails string, filter paper, drawing pins, thread. By cutting through a cone at different angles to the base we can produce a family of interesting geometric curves are called conic sections. These curves are circle, parabola, hyperbola and ellipse. Conic sections using modeling clay: If a hollow cone is cut open by cutting along its slant edge and unrolled, we get a sector of a circle. Hence a cone can be rolled from a portion of circular paper. Cut a piece of chart paper into a circle using a divider. Cut away approximately a quarter of a circle by cutting along radial lines, retaining three quarters of it. This can be rolled into the shape of a cone, and the overlapping portion can be pasted or stapled together. Now make an identical cone in the same way from butter paper or tracing paper, but do not staple or paste it into the final shape. Place it inside the chart paper cone to form an inside layer. Now fill the cone with modeling clay. Press it down till it take a shape of a cone. Let the clay cone with the butter paper cover fall out

of the paper cone. Peel off the butter paper. The cone is now ready for cutting. Stretch a thin wire holding the ends with both hands till it is taut. Use this wire to cut the cone at different angles to the base.

- A cut parallel to the base gives a circle at the section.
- A cut parallel to a sloping side gives a parabola.
- An ellipse is made by cutting through the cone at a slant.
- A cut parallel to the axis of the cone gives a hyperbola.

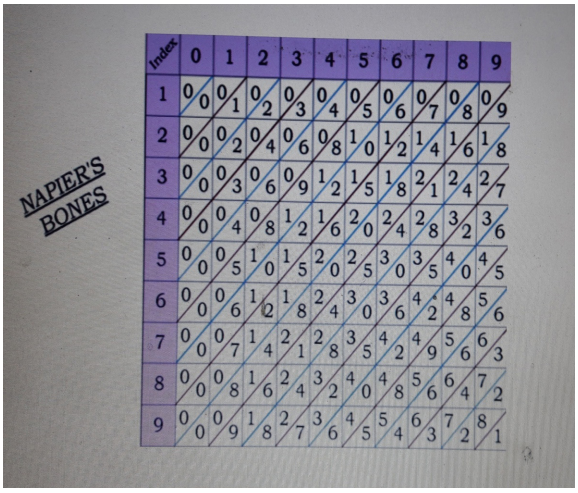
Napiers Bones

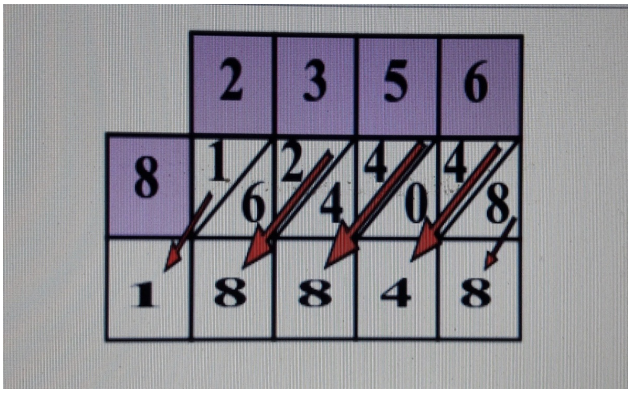
Materials required: Cardboard or chart paper. John Napier (1550-1617), the man who invented the logarithmic tables also made this simple calculation device. Napier's bones can be used for multiplying a large number with a single digit number. In this way



it reduces multiplication to a series of additions. Take nine long strips of chart paper or cardboard, 1 inch wide and 9 inches long. Copy the numbers shown in the figure on the nine strips in the manner shown along with the diagonal lines. Now, to multiply 2356 by 8, take the bones with the digits 2, 3, 5 and 6 and lay them next to each other. On the 8th horizontal row add the digits that are formed along each diagonal to get each digit of the answer. The method of adding along the diagonals used here is very similar to the Gelosia method of multiplication developed by Indian mathematicians many centuries ago. It is also quite similar to the method of multiplication taught at school. On observing closely one finds that each of Napier's bones is simply the multiplication table for that number (verify this from the figure). The digits in the units place and the tens place are written separately. So while using Napier's bones, one need not make the effort of recalling multiplication tables. Addition of digits, carrying over the digit in the tens place are managed by the diagonal arrangement.

Take nine long strips of chart paper or cardboard, 1 inch wide and 9 inches long. Copy the numbers shown in the figure on the nine strips in the manner shown along with the diagonal lines. Now, to multiply 2356 by 8, take the bones with the digits 2, 3, 5 and 6 and lay them next to each other. On the 8th horizontal row add the digits that are formed along each diagonal to get each digit of the answer. The method of adding along the diagonals





used here is very similar to the Gelosia method of multiplication developed by Indian mathematicians many centuries ago. It is also quite similar to the method of multiplication taught at school. On observing closely one finds that each of Napier's bones is simply the multiplication table for that number (verify this from the figure). The digits in the units place and the tens place are written separately. So while using Napier's bones, one need not make the effort of recalling multiplication tables. Addition of digits, carrying over the digit in the tens place are managed by the diagonal arrangement.

Reference :

Voderman, C., How Mathematics Works, Kindersley, D., London(1996):23

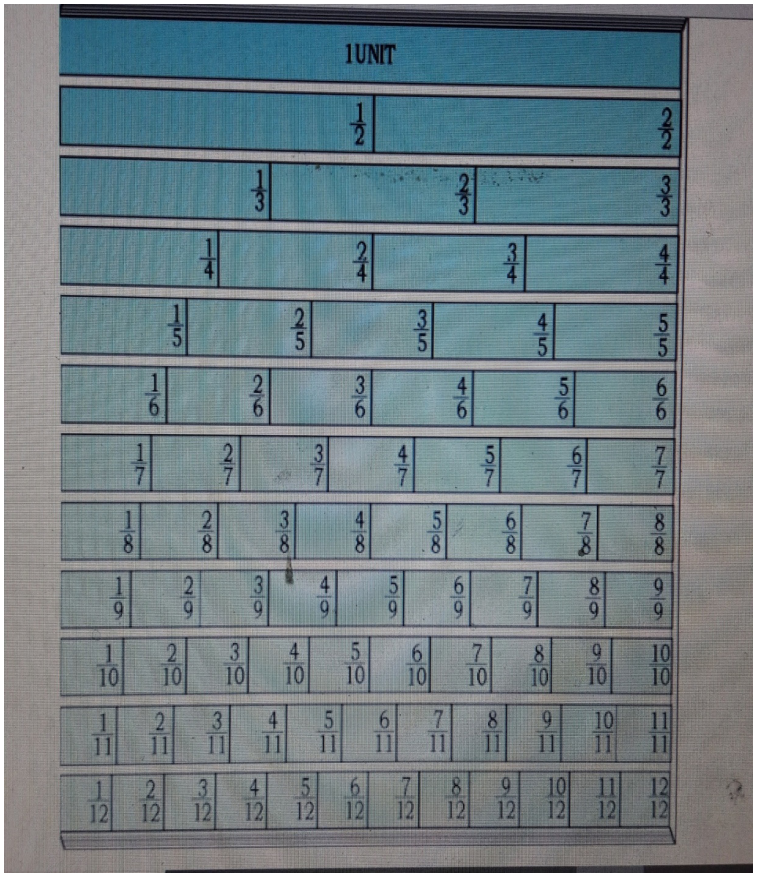
GCD of Two Numbers

Materials required: a piece of paper, a scale and a pencil. The G.C.D. of a pair of integers can be found by an interesting geometric method. (G.C.D. of a set of integers is the largest integer which can divide all the numbers without remainder). In order to do this, we only need a piece of paper, a scale and a pencil. Suppose the two integers whose G.C.D. is to be found are a and b. First draw a rectangle on the paper of length a and breadth b. (If a and b are large take the length and breadth as a /2 and b/ 2 , or in some suitable proportion). From this rectangle mark off the largest possible square. If a is greater than b, this will be a square of side b. After marking off the square, the portion which remains is a rectangle with sides b and a – b. Again mark off the largest possible square from this rectangle. Continue this process till you obtain a square instead of a rectangle. The measure of the side of this square is equal to the G.C.D. of the original pair of numbers. This method is based on the fact that if a and b are both divisible by a number, then a – b will also be divisible by the same number. The same principle is applied recursively to obtain the G.C.D.

Fraction chart (A teaching aid)

Materials required: Plywood or cardboard, Chart paper, string, and bob The concept of fractions of a whole is introduced in primary school. Students find it difficult to master and often even understand the concept of a fraction and the meaning of the numerator and denominator. A fraction chart is a very useful teaching aid which can be used for this purpose.

A fraction chart is made from a piece of plywood or thick cardboard which is large enough to be put up on the wall. Narrow strips of chart paper of equal length are pasted on the board at equal distances. Let the first strip represent 1. Divide the next strip into two equal halves and mark the fractions 1 2 and 2 2 . Divide the next strip into three equal parts and mark the fractions 1 3 , 2 3 and 3 3 . Continue in this way till all the strips are divided to obtain smaller and smaller fractions. One can make a chart till the fractions 1 20 , 2 20 . . . if space is available. Now suspend two long strings from the top of the board with bobs attached at the end. The strings remains vertical like a plumbline. The fraction chart can be used for showing the part whole relationship: how many one thirds make up one? Another important use of the chart is to show equivalent fractions. Drop the plumbline over a fraction, and if the chart is aligned vertically all the fractions which coincide with the plumbline are equivalent fractions. The students also learn that any

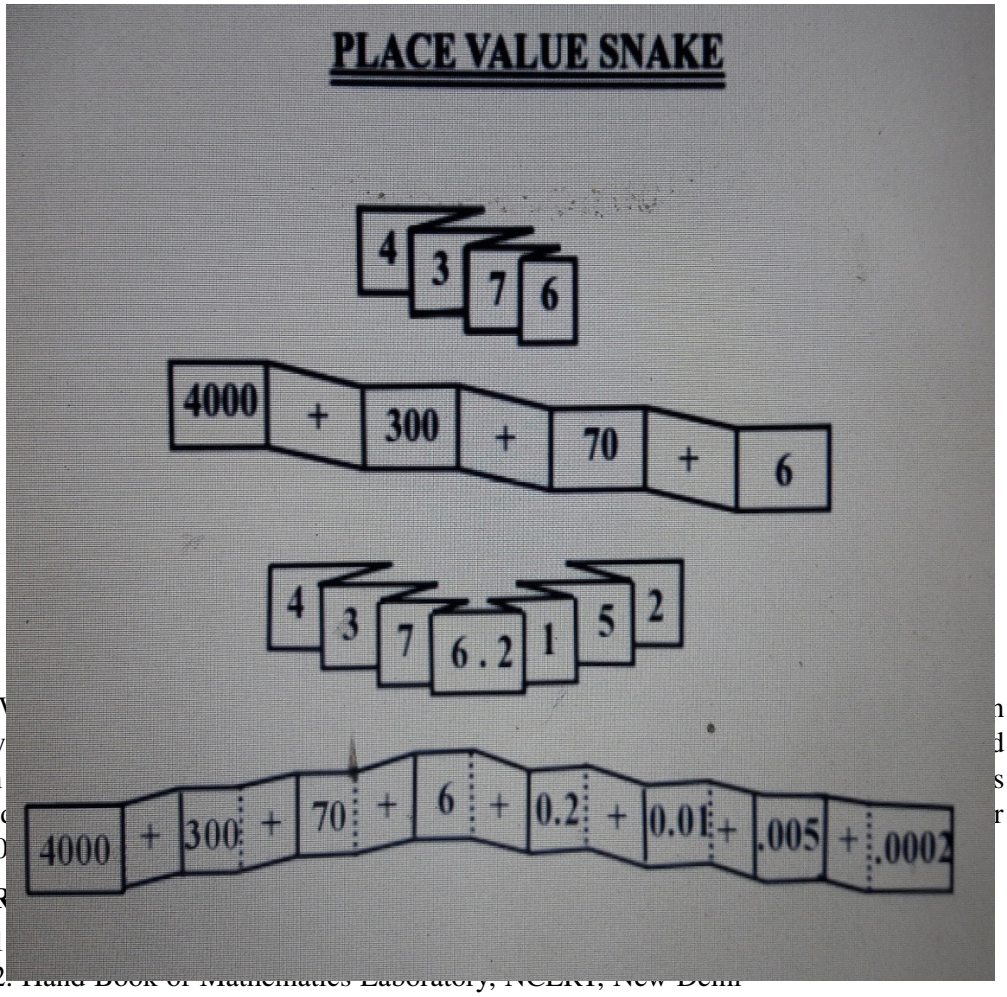


fraction of the form n n is equal to 1. It is also possible to do some rudimentary addition and subtraction of fractions with the chart. If two fractions are to be added find their equivalent fractions on the same line of the chart by dropping the plumbline. Now it is possible to add the fractions easily by adding the numerators.

Place value snake

Materials required: Strip of paper, pen The concept of place value is not easy to grasp for very young children.

The place-value snake is a delightful activity which helps in fixing the concept in the mind of a young child. To make a place value snake, take a strip of paper and fold it in the manner shown in the figure.



2. Hand Book of Mathematics Laboratory, NCERT, New Delhi

Article for Prof. S. N. Giri Memorial Volume

Bhaskaracharya : A Great Teacher

Sudhakar C. Agarkar

Introduction

I am pleased to learn that a Memorial Volume is being brought out in the memory of Professor Giri. He was a great teacher and created interest the minds of a large number of students towards mathematics. As a tribute to this great soul I wish to write a life story of yet another geat Indian mathematician: Bhaskaracharya. The present article describes the salient features of his writings highlighting the notable pedagogic aspects embedded in it. There were two Shankaracharyas in the history of mathematics. I am referring to Bhaskara-II who contributed to mathematics in 12th century.

Bhaskara

As per the historical records Bhaskara-II was born at a place called Bijjal Bid close to Sahyadri ranges. Researchers believe that this place was close to Chalisgaon in the state of Maharashtra. A stone carving found near *Patnadevi* describes how knowledgeable he was. It states that Bhaskaracharya had studied grammar, *natyashashtra* and medicine apart from mathematics. The carving also mentions that he wrote Siddhanta Shiromani at the age of 36. The recorded year of his birth is *shalivahan shaka* 1036. Since there is a difference of 78 years between Salivahan calendar and Gregorian calendar the birth year of Bhaskaracharya as per Christian era comes to be 1114. He was a prolific writer. He has written so much and I confess that my life is too short to read all his writings. Instead, I will concentrate my attention on his famous book “Lilavati”.

Lilavati

Lilavati, sometimes also called as patiganit, mainly discusses concepts in arithmetic. In addition, there are elementary concepts taken from Algebra and Geometry also. The entire book is written in a poetic from. The capability of Bhaskaracharya as a litterature is evident from the way he puts down the verses (shlokas) in the book. He has made profuse use of alankaras (figures of speech) like pun, alliteration, metaphors and similies in his writing. There are indications that he also had a good sense of humour. For example, in one of his problems in Lilavati a rich man gives so many fractions of coins for many days to the beggar thta the total is just one kawadee.

There are about 270 verses in Lilavati. Only a few of them provide guidelines white a majority are used to raise questions. The book begins with an invocation addressed to Lord Ganesha seeking his blessings. I would like to quote it in Sanskrit with English translation by H T Colebrook

प्रीतिभक्तजनस्य यो जनयते विघ्नं विनिघ्नन स्मृतः।
तं वृंदारकवंद्रदितपदं नत्वा मतंगाननम॥

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पाटीसद्गणितस्य वच्मि चतुरप्रीतिप्रदां प्रस्फुटाम।
संक्षिप्ताक्षरकोमलामलपदैर्ललित्यलीलवतीम॥

Having bowed to the deity, whose head is like an elephant; whose feet are adorned by gods; who, when called to mind, relieves his votaries from embarrassment, and bestows happiness on his worshippers; I propound this easy process of computation, delightful by its elegance, perspicuous with words concise, soft and correct, and pleasing to the learned.

There is a famous legend associated with the book Lilavati. It is said that Lilavati was the name of Bhaskaracharya's daughter. He was not only a mathematician but also an astrologer. Looking at the horoscope of Lilavati he realised that her married life could be problematic which could be avoided by marrying her at a particular *muhurta*. To ensure that the marriage takes place on that particular *muhurta* he prepared a *ghatika patra* and left it in water container. It was so adjusted that the marriage ceremony was to begin as the *patra* dipped under water. Impatient and curious Lilavati went to see how far it dipped. While she was standing close to the container a small bid from her ornament fell down and blocked the hole of the containr. As a result, the water flow slowed down and the *ghatika patra* dipped later than it should have in normal circumstances. Thus, the marriage ceremony took place later than its planned time.

As the destiny had deciede Lilavati became widow and came back to her father after a few years of her marriage. We know that remarriage was not permitted in those days. Hence, widowed Lilavati would remain sad and silent. In order to keep her engaged Bhaskaracharya used to give her interesting problems. As Lilavati was quite intelligent she could solve them quickly. It is stated that the discussion that took place between father and daughter is presented in the form if a book entitled Lilavati.

Researh into the genesis of Lilavati does not support the above legend. Nonetheless, the picture depcting the interaction between an old man and a young lady is seen at various locations especially on the cover page of the Persian translation of Lilavati done by Shri Faizi in 1587 who worked in the *darbar* of king Akbar, the great. Since then the story has been spreading in all parts of the country and is accepted as a fact.

Pedagogical Characteristics

Lilavati advocated many useful pedagogical aspects that help the teaching of mathematics. Looking at Lilavati through this angle as a practising mathematics educator I see following pedagogic characteristics of Lilavati.

Gender Equality

Whether the legend stated above is true or not on thing is clear that Bhaskaracharya advocated gender equality through his writings. At many places in Lilavati he refers to readers as hale, sakhe, charming girl, etc. One must keep in mind that those were the days when girls were discouraged from formal schooling. Even when the education has been opened to female members they opted for softer subjects and avoided hard disciplines like mathematics. It was certainly revolutionary on the part of Bhaskaracharya to address mathematical problems to a female member through his writings. An example in this case

would support what I am saying.

बाले बालकुरंगलोलनयने लीलावति प्रोच्यताम।
पंक्तत्क्षेकमिता दिवाकरगुणा अंकः कति स्यूर्यदि॥
रॉअॉस्यनविभागखंडगुणाने कल्यासि कल्याणिानि।
थिन्नस्तेन गुणोन ते च गुणिाता अंकाः कति स्युर्वद॥

A beautiful and dear Lilavati, whose eyes are like fawn's tell me the numbers resulting from one hundred thirty five taken into twelve, if thou be skilled in multiplication by whole or parts whether by subdivision of form or separation of digits.

Building Prerequisities

Mathematics is a hierarchical subject. All that is learned earlier is essential to progress further in mathematics. If a student does not possess requisite knowledge or skills he/she would not be able to handle new situation. From the sequence of examples in Lilavati it is clear that Bhaskaracharya makes deliberate efforts to build essential prerequisites. The examples given under a specific topic are so graded that students develop all the necessary prereqisites knowledge and skills before reaching a major problem in the section. This helps them build confidence and deal with the main problems without any difficulty.

Motivation

Motivation is considered to be an essential component of learning. Unless the person is motivated the learning would not take place even if the teacher has completed teaching. Importance is, therefore, given in school teaching to motivate the learner even before a task is given. Bhaskaracharya seems to be aware of this pedagogical fact. Hence, he tries to create interest among the readers trough a vareity of means. In some cases he presents the problem through a story. In other cases he creates interesting situations. A problem involving snake and peacock can be considered as an example in this category.

अस्ति स्तंभतले बिलं तदुपरि क्रिडाशिखंडी स्थितः।
स्तम्भेहस्तनवोच्छिते त्रिगुणितस्तंभप्रमाणांतरे॥
दृष्टवाहिं बिलमाव्रजंतमपतत तिर्यक स तस्योपरि।
क्षिप्रं ब्रूहितयोर्बिलात्कलिमितैः साम्येन गत्योर्युतिः॥

A snake hole is at the foot of a pillar, nine cubits high, and a peacock is perched on its summit. Seeing the snake at a distance of thrice the pillar gliding towards the hole, he pounces obilquely upon him. Say quckly how many cubits from the snake's hole they meet, both proceedings an equal distance.

Rapport Building

Most of the times the book on mathematics prescribed for formal study is written in a dull and dry style. The entire book Lilavati is, however, written in first person active voice. The author addresses the reader with loving words like mitra, bale, sakhe, etc. At many places he addresses the readers as mathematician and instigates them to undertake

problem solving.

Open enddedness is the hallmark of Lilavati. Bhaskaracharya has not solved any problem in his book. He suggests multiple ways of dealing with a problem and allows the reader to choose the method of his or her choice. At some plaes he provides necessary hints but does not force the reader to follow a specific method. Here is an example from Lilavati as a part of kuttak method.

एकविंशतियुतं शतद्वम। यद्गुरां गराक पंचषष्टियुक॥
पंच्वर्जितशतद्वयोध्दतम। शुद्ध्दिमेति गुराकं बदशु तत॥

Say quickly, mathematician, what is that multiplier, by which two hundred and twenty-one being multiplied, and sixty-five added to the product, the sum divided by a hundred and ninety-five becomes exhausted.

Content Relevance

Studies show that students dislike mathematics as they find it irrelevant to their daily lives. Realising this fact Bhaskaracharya attempts to pose problems of daily relevance like income, area, stacking of bricks, etc. The probl;ems are composed taking animate as well as inanimate objects found around. Thus snakes, bees, swans, elephants, etc. are referred to in the description. Apart from showing the relevance these problems create interest among the students for problem solving. Here is one such example from Lilavati.

यातं हंसकुलस्य मूलदशकं मेघागमे मानसं।
प्रोड्डीय स्यलपद्मिनीवनमगदष्टांशको भस्तटात॥
बालेबाल्मृणालशालिनि जले केलिक्रियालालसम।
दृष्टंहंसयुगत्रयं च सकलां युथस्य संख्याबद॥

Out of the swans in a certain lake, ten times the square root of their number went away to Manasa Sarovara when rains started, and one eight the number went away to the forest Sthala Padmini. Three pairs of swans remained in the tank, engaged in water sports. What is the total number of swans?

Field Experiences

In order to popularise Lilavati among students the Vidya Prasarak Mandal (VPM), Thane has deciede to organise workshops. These workshops were conducted in different schools and colleges in the states of Maharashtra, Madhya Pradesh, Rajasthan and Andhra Pradesh. Each workshop is conducted in two parts. The first part is devoted to familiarising the participants with the rich tradition of Mathematics in India. Bhaskaracharya is presented as a member of this tradition. Some information about his life and work is outlined. The second part of the workshop is used to deal with selected problmes from Lilavati. These problems are chosen taking into account the educational background of the participants.

The experiances of conducting these workshops have been quite positive. Students,

at different levels, find the task of solving problems from Lilavati very interesting. It is notable that the participants engage themselves in the task for almost three hours. On many occassions they come forward to solve problems on the blackboard. Due to educational practice that is followed in the school they do not appreciate the open ended approach advocated by Bhaskaracharya to begin with. Soon, however, they realise the importance of this method and come out ith novel methods of dealing with given problem. Let me share an experience that I had while solving the snake and peocock problem mentioned above. Standard method that is followed in dealing with the problem is to apply Pythagoras theorem and find out the value of requisite distance. One boy however, came out with 12 as the answer quickly. On enquiry he revealed that he tried to look for a triad having number 9 in it. Another boy went on adding square numbers to 81 (square of 9) and checked whether the answer is a square number. Through this method he also came out with an answer 12.

Conclusion and Implications

It is clear that the pedagogical Characteristics of Lilavati are relevant even today. I suggest that the efforts must be made to practice them in day to day teaching. I believe that the methods adopted by Bhaskaracharya would work both for first generation learners and traditional learners. It would, I am sure, enable us to make indian citizens mathematially literate. We should be proud that we have an age old tradition of teaching mathematics effectively. It is high time that we make use of this traditional knowledge to improve teaching of mathematics in Indian schools and colleges. Constructivistic approach, situational learning and experiential learning are the buzz words of present day pedagogy. Without saying it explicitly, Bhaskaracharya had adoped all of these pedagogic techniques in his writings. The very meaning of his name is Bhaskar the teacher (Acharya in Sanskrit means teacher). I once again salute this great teacher of India.

Gateways to Strengthen Mathematical Thinking for School Students to Materialize NEP 2020

Arup Kundu*

Abstract

This paper enlightens some gateways for teaching and learning in mathematics of the school students. Generally, teachers focus their teaching processes on factual knowledge rather than conceptual clarity. The National Achievement Survey (2017) reported continuous degradation in achievement in mathematics with increasing order of grades across the India. It is also apparent that most of the students loosen their interest in mathematics up to secondary level (NCF, 2005). NEP (2020) proposed re-shaping in curriculum and pedagogy and recommends the paradigm shift from content based to experience based learning. The NEP emphasizes on strengthening the foundation of quality education. It focuses on cognitive depth of the learner as an important aspect and on restrains the rote learning system. The gateways which are discussed in this paper can be very useful for the school students and teachers in mathematics teaching learning process.

Key Words: Gateways, teaching, learning, mathematics, NEP2020.

Introduction:

Education is an important vehicle for individual and social development. Defective education hinders development. Our education system needs a reform today. The written system is considered as an important component in the Indian education system. To ensure the pass percentage in this test, students often try to memorize without trying to understand the concept of the subject. Report cards may show a decent position in school or board results but this education will not be successful in real life. The new national education policy has come up with new ideas to solve such problems.

*Restructuring school curriculum and pedagogy:*The new policy known as NPE 2020 recommends a 5+3+3+4 structure instead of the conventional 10+2 school system. In this new composition, three years of 'Anganwadi' or play school will be added to twelve years of regular school as it is important for the development of mental capacity in children aged 3-6 years. There are two sections to this foundation stage, as recommended in NPE 2020. Compromise of the first part of 3 years of play school and only elementary classes in the second segment consists of one and two grades. The foundation stage will combine plays / activity based learning and the aforementioned ECCE(Early Childhood Care Education) curriculum and pedagogy. The remaining ten years are preparatory stage (grades 3 to 5), middle school (grades 6-8) and secondary stage (grades 9-12) till 18 years of age. The preparatory stage will gradually transform from play based learning to a more formal but interactive classroom learning with the introduction of some (light) textbooks that include reading, writing, speaking, physical education, a strong focus on various languages, science, and mathematics including art the foundation can be laid. The middle stage students will see the role of subject teacher in learning /discussing more

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abstract concepts in each subject across intermediate level sciences, mathematics, arts, social sciences and humanities for which students will be prepared at this stage. The high school (or secondary) level will consists of four years of multi-disciplinary study, based on medium –level subject-based teaching and learning styles, but with greater depth, greater focus on life aspirations, and greater flexibility. Emphasis will be placed on student preferences. Although investing in Fundamental literacy and Numeracy can increases the cost of education, the quality of education needs to be improved. Fundamental literacy and Numeracy in primary education requires the inclusion of appropriate skills in language and mathematics, to increases students' learning levels ad to reduce dropout rates in after primary school.

Importance of Mathematics in perspective of NPE 2020: Mathematics is an important part of the curriculum at every stage of school education due to its numerous applications in all walks of life. The National Policy on Education (NPE, 1986) stated that "Mathematics should be visualized as the vehicle to train a child to think, reason, analyze and to articulate logically". Similarly, the new National Education Policy (NEP, 2020) recognized importance of mathematics and mathematical thinking in upcoming research –oriented fields such as artificial intelligence, machine learning and data science. Mathematics is considered to be a science of mathematics and computation surrounded by the tyranny of a correct answer where in the broadest sense it is a matter of conjecture which makes logical conclusions. It is the study of abstract systems created by abstract elements. Mathematics is not just a science of numbers or calculation but much more about generalization formation, relationship viewing, logical thinking and the development of reasoning. Many students struggle with it and become dis-satisfied because they are constantly interrupted in their engagement (Anthony &Walshaw, 2009). The National Focus Group on mathematics Education pointed out the problems of mathematics education in 2005 and made some recommendations in reference to the curriculum, pedagogy and classroom environment. Although the paradigm shift was recommended in 2005 in all important aspects of the teaching learning process, the effectiveness of mathematics is deteriorating (National Achievement Survey, 2017) across the country.In November 2017, National Achievement Survey was conducted across the country for third, fifth and eighth grades in government and government aided schools. At the national level, the percentage of correct answers in mathematics was 63% in the third grade, 53% in the fifth grade and 43% in eighth grade. There are reports of a gradual decline in performance with increasing grades. It is also clear that most students lose their interest in mathematics up to secondary level and deliberately choose a stream without mathematics. There are many reasons behind this negative attitude of students towards mathematics such as weak mathematical foundation, irregular study habits, parental noninvolvement etc. (Vijyana, 2014). In addition to these topics, teaching methods, strategies, techniques and the classroom environment also contribute to developing a positive or negative attitude towards the subject. Typically, teachers adopt such teaching methods, strategies and methods in the mathematics classroom that focus on methodological knowledge rather than conceptual clarity. Explanatory methods, including oral and verbal, that give all the importance to lectures and lessons, are used by teachers

in the mathematics classroom(Khongji&Nongbsap, 2013). These methods are content centered where students have to memorize the mathematical rules presented earlier without any engagement(Khongji&Nongbsap, 2013). Relevance to the content and engagement with previous experience is essential to making learning meaningful (NEP, 2020). The NCF, 2005 promoted the paradigm shift in educational practice to allay fears and feelings of failure. It should be designed to make this subject enjoyable rather than burdensome. It should be more interactive and involve collaborative and exploratory activities that will create fun instead of fear. NEP(2020) proposes a paradigm shift from content based learning to experience based learning to make learning meaningful and engaging. This article highlights the approaches, techniques and methods of teaching mathematics that can be helpful in bringing innovation to the mathematics classroom.

Gateways to Enhance Mathematical conception in Mathematics classroom:

There are enough ways to teach and learn mathematics, and each method is unique in evoking mathematical thinking. Before unveiling the gateways, one must understand the human information processing system. In the context of young students, the main features of the system are involved:

Learner is the constructer of knowledge rather mere receiver:

The learners are actively involved, the environment is democratic, the activities are interactive and student centered, the teacher facilitates a process of learning in which students are encouraged to be responsible and autonomous.

Experience leads to learning:

Learning by induction is the key process for learning of a human being.

Learning that encourages students to engage in problem solving and experiential learning:

A learning process that engages students by connecting to the real world through exploration and high level questions. For example, during an inquiry based lesson the teacher can see how each student sees a problem and what steps they take to find a solution in their own unique way.

The human being have a limited capacity of working memory *i.e*. More focus on understanding and skill acquisition and less focus on memory.

Enable children to adjust to their own pace of learning: Student who needs extra time may spend more time practicing until they understand where students with less ability can move on. It frees the teacher to help children who need more help at the individual level.

Focus on learning how to learn, not just learning i.e. meta-cognitive abilities of the human needs to be explored.

The method can either have steps or features or the method present an idea that teachers can share to motivate the learning process. In the present study, an idea has been presented in some methods and the responsibility of formulating a plan using innovative efforts rests with the teacher whereas in some methods the steps and features are discussed in detail. Appropriate gateways to the NEP 2020 approaches for young students are discussed below:

Constructivist Approach: Position paper national focus group on teaching of

mathematics, recommended an approach in 2005 that facilitate learning of students i.e. Constructivist approach. Most of the teachers in the school are not aware of this method till now. NEP (2020) also emphasized on such approach which promotes higher order thinking instead of rote memorization. Research from different parts of the world reveals that constructive approaches are a successful way to provide students with meaningful learning experiences in a classroom (Brooks & Brooks, 1999; Johnson & Johnson, 1994). The constructive approach is believed to increase children's creativity, increase interest in mathematics and is considered more effective thantraditional methods in developing social skills (Nayak&Senapaty, 2011, McCray, 2007). The constructivist approach radically changes the process of teaching and learning mathematics, integrating it into everyday life, instead of just teaching abstract (Vinetere, 2018). In a constructivist classroom, teachers create situations where students will question their own and each other's assumptions. Similarly, a constructivist teacher creates a situation where he is able to challenge the assumptions on which traditional teaching and learning are based. It is a learner centered approach where the student is not merely a receiver but a creator of knowledge. Teacher acts as a facilitator and provide students with experiences that allow them to guess, predict and manipulate mathematical information. Under this approach tasks such as research, investigation, imagination, innovation and questioning are performed by the student (Gray, 1997). The student's own intuitive mathematical thinking gradually becomes more abstract and potent through mathematical work and interaction with other students(Clements & Battista, 2009 cited in Vintere, 2018).Constructive approach based classrooms are distinguished from a conventional classroom by identifiable qualities such as student activism, democratic environment, interactive, and student-centered activities. In constructive classrooms, students are encouraged to create their own ideas based on their prior knowledge and experiences. Observations like best learning are possible when the student is trying to understand the concept through prior experience (Sims, 2002).Hmelo-Silver, Duncan, and China (2007) emphasizes the importance of constructive approaches to problem based and inquiry learning methods for learning science in a better and more effective way.Constructivist approach has a positive effect on students' achievement in mathematics as well as helps to develop a positive attitude towards mathematics (Clarke, 1997 and Simon &Schifter, 1997). There are various teaching strategies that follow the principles of constructive approach such as Inquiry Based Learning, problem based learning, experiential learning etc. These strategies must be utilized by the teachers to bring innovations in classroom. Teacher must use these techniques to innovate in the classroom.

5 E's Learning Model:

The 5 E's Learning Model comprises five phases:

Engagement Phase: Learners are engaged in any task in any form of classroom where it serves as an opportunity for students to build relationship between previous knowledge and existing concepts.

Exploration Phase: Learners discuss and explore in groups, engaging in events and materials. They create a platform for general experience and therefore a vast field of experience.

Explanation Phase: Students with general experience begin to lay the groundwork

for abstract experience and to clarify their miscommunication under the teacher's interpretation.

Elaboration Phase: Students describe their knowledge in detail and therefore expand their knowledge. In fact, students apply the acquired ideas to connect with other related fields and therefore gain real world perceptions.

***Evaluation Phase*:** In evaluation stage determines whether the student has acquired the necessary knowledge and perception.

Experiential Learning

Mathematics is undoubtedly a difficult subject. Even great minds like Albert Einstein know that there are difficulties in learning. It is not surprising that mathematics teachers have difficulty teaching students. The lecture system where teachers allow students to memorize mathematical information is long gone. Today, teachers are called upon to teach new and effective teaching methods so that not only master, but also boot comprehension is developed. The NPE 2020 proposed the adoption of experiential learning for all levels of school education. Mathematics requires experiential learning where students engage with their own understanding of mathematical concepts and practices. Experiential learning is the practice of learning by doing. It encourages students to gain direct experience with materials rather than learning through someone else's experience in textbooks and lectures. Experiential learning focuses on individual learning. It is defined as learning through reflection. It involves one more step through the reflection of other strategies such as activity based, problem based and comparison with discovery learning. Experiential learning can be helpful in subjects such as mathematics where students engage with their own understanding of mathematical concepts and practices. Introducing experiential learning activities in mathematics classrooms is a way to create innovative classroom environments that help students overcome mathematics anxiety. Activities need to be linked to teachers' ideas so that students can engage and reflect on their actions e.g. if the teacher in primary school wants to teach the duration and time of sunrise and sunset. He will ask them to prepare a chart of the rising and setting of the sun for a few days which will give them a deeper idea about the duration of a day rather than telling them directly. An activity related to the sale of self-made food items in secondary school that involves purchasing, measuring ingredients and reflecting the process they undertook for all the activities involved. The concept of probability in secondary school can be taught through experiential learning involving activities related to concrete items such as coins, cards, and dices.

Interpretation Construction Design Model (ICON)

The teaching and learning methods of the ICON model emphasize the students face to face authentic problems in groups when it comes to constructing explanations or searching for information groups and encountering different interpretations of the problem in the group; the process of individual learning is an evidence and therefore dual purpose education is an acceptable model.

It is a model which contains seven steps:

Step 1*Observations:* Students observe the situation or problem before proceeding to solve it.

Step 2: *Interpretation Construction*: Students relate the situation or problem to previous experience.

Step 3: *Contextualization*: Students collaborate to discuss in groups and explore to understand their analysis and interpretations.

Step 4: *Cognitive Apprenticeship*: Students collaborate to discuss in groups and explore to understand their analysis and interpretation.

Step 5: *Collaboration:* Students analyze the knowledge that is created and create an explanation of their own.

Step 6: *Multiple Interpretations*: Students use interpreted knowledge and multiple interpretations are given by them instead of solving problems.

Step 7: *Multiple Manifestations*: Students apply multiple interpretations to get multiple solutions to the problem.

Technology Enabled Learning (TEL): To expand and improve student centered learning by describing the use of technology, platforms, systems and digital content. Technology reduces human efforts and increases efficiency in all areas of life. Education is no exception where technology has entered and teaching is contributing to increasing the efficiency of the learning process. There are verity of technology enabled learning strategies that can be categorized based on hardware, software, and utility objectives. Web-quest learning, M-Learning and Blended Learning are just some of the things that must be practiced in a mathematics classroom to make learning interesting and joyful.

Web Quest Learning: Web Quest learning is knowledge with the help of internet resources. A Web Quest is an Inquiry Based Activity in which all or most of the information use comes from internet resources."A study on teaching how to learn, a research –oriented activity those requiring students' interactions and providing knowledge from sources on the internet completely or a certain extent or via video conferencing."(Dodge, 1997).

According to the creators of Web Quest, a Web Quest should consist of six main sections, introduction, task, process, resources, evaluation and conclusion.

*Introduction:*The introductory part will focus on the student's subject and what he or she is waiting for, stimulate the student's interest in a variety of ways, determine the situation, and provide an initial overview of the situation. Also, in this chapter, the problem of the core center of web quest is also introduced to the students.It should be interesting, inspiring, original, and of course guided by the student. At the end of the activity, students should have a description of what they should do.

Task: Part of the task should be interesting, inspiring, competent and authoritative and should be conducted by the student and at the end of the activity the students should include a description of what they did and include a verbal presentation or a product.

***Resources*:** This section includes the website address assigned by the teacher to help the student complete the process. It may contain sources other than websites. No one can use all the resources. This may include the links needed to perform the operation.

Process: It consists of process steps that are clearly defined. It provides a review

of the process that needs to be done to achieve the operation. This may also include recommendations for learning. The steps that students need to perform should be presented to the students in clear and concise steps.

Evaluation: This section includes how the information collected will be organized, how the results will be evaluated, and what the criteria for evaluation will be. In general, the grading is evaluated using a scale (rubric).

Conclusion: This is the stage to remind students of what they have learned and what they have managed or finished web queries. Students are encouraged to expand their experience in other fields. Thoughts about results and achievements are shared with students.

It is used to stop web adventures. In this section, students can find summaries of information about what they have learned and what they have achieved. They are encouraged to expand their experience in other areas.

Blended Learning

Blended learning is not only a combination of online and face to face modes, but also a well-planned one and a combination of meaningful activities in both modes. The mix demands consideration of a variety of factors, focusing primarily on learning outcomes and the student-centered learning environment.

Role of Learner in the Blended Learning Environment:-

***Increase student interest*:** When technology is integrated into school lessons, students are more likely to be interested, concentrated and excited about the subject studying.

Keeps students focused for longer:The use of computers to search for information and data together is a tremendous life saver including access to resources such as the internet to conduct research. This engagement and interaction with resources keeps students focused for a long time then they will be with books or paper resources, this engagement also helps in the development of learning exploration and research.

*Provides student autonomy:*The use of e-learning materials enhances a student's ability to determine appropriate education take responsibility for the goal and his own learning, which develops a power which will be translatable across all subjects.

Instill a disposition of self-advocacy:Students become self-driven and responsible, tracking their personal achievement, which helps them develop the ability to find resources or get the help they need, self-advocacy so they can reach their goals.

*Promote student ownership:*Blended Learning evokes a sense of 'student ownership over learning' which can be a powerful force driving learning; it is this sense of responsibility that helps to feel ownership.

Allow instant diagnostic information and student feedback: Gives teachers the ability to quickly analyze, review, and respond to students' work, the teacher has the ability to create his own teaching methods and responses for each student improving time efficiency.

*Enables students to learn at their own pace:*Due to the flexibility of Blended Learning and the ability to access internet resources allow students learn at their own pace, which means a teacher can help speed up the learning process or provide more advance resources if needed.

Prepare students for the future: Blended Learning offers many real-world skills that

translate directly into life skills(Blended Mode of Teaching and Learning: Concept Note-UGC):

Research Skills
Self-learning
Self-engagement
Helps to develop a 'self-driving force'
Decision making
Provide a greater sense of responsibility
Computer literacy

Role of teachers in Blended Learning Environment:-

Blended learning takes the role of teacher from knowledge provider to trainer and mentor. This change does not mean that teachers play a passive or less important role in students 'education. In contrast, with –Blended learning, teachers can have a deeper impact on students' learning.

Lin, Tseng and Chiang (2016) conducted an experimental study to see the effectiveness of Blended Learning on high school mathematics students in Taiwan. They have not only had a positive effect on learning outcomes but have also changed attitudes towards mathematics. Results have been found in favor of Blended learning in mathematics classrooms with reference to achievement and attitude (Awodeyi, Akpan&Udo, 2014; Abramovitz, Berezina, 2012). Generally, Moodle learning is used for such learning environment. Teacher can conduct interactive activities for online group discussion, testing and evaluation. This strategy provides flexibility in the context of time and space.

M-Learning:

M-Learning is a strategy where learning occurs through multiple interactions, technical, social and content interactions. "Mobile learning is a type of learning whose learners is determined previously, is not in a specific location, or benefits the opportunities offered by mobile technologies" (O' Malley, Vavoula, Glew, Taylor, Sharples&Lefrere, 2003). In our time, the development of information technology and technical devices in education has accelerated. The information technologies used in education have advanced rapidly and reliably in such a way that traditional teaching methods have lost their importance in technological education systems. This advancement has revealed the concept of e–learning. With the help of today's mobile technology in e-learning in the concept of d-learning (distance learning), the concept of m-learning has provided technological advancement in education. The development of mobile technology and the need to take education technology to a new level has revealed a new concept of M-Learning. The most important advantage of e –learning from M-learning is that students can demand information independently from time and environment. Mobile learning can be used to support traditional learning (Wang, 2004) as well as distance learning (Mutlu, M.E& others, Barbara et al., 2005).

M-Learning techniques are available using personal electronic devices such as handheld computers, notebooks, mobile phones and tablets. Supandiet. Al. (2017) and Etcuban&Pantinople (2018) has found that mathematics education supported by mobile

phone applications improves the achievement of school students. In addition to web based learning, many mobile applications such as Socratic, Photomath, 'Myscript Calculator 2' etc are commonly used by students. "Socratic" is a free photo based software that provides step by step solutions to mathematics problems. 'Myscript Calculator 2' is an app that is more than a calculator. It covers the handwriting into text and then solves the problem. The app includes support for fundamental operations like addition and subtraction, power, root, index, trigonometry, logarithm, constant (e.g. Pi) and much more. Also various learning management software such as Google Classroom, Moodle, Google meet, EDMODO are now popular for connecting with students at any time.

If we analyze mobile education in terms of its benefits, we can classify them as follows:

Lifelong learning
Learning inadvertently
Learning in the time of need
Learning independent of time and location
Learning adjusted according to location and circumstances (Bulun& others, 2004).

Conclusion:

The various view of NEP2020 regarding the subject of mathematics teaching and learning impact directly on teachers. NEP 2020 has suggested a clear pathway which will inculcate conceptual lucidity as well as creativity and critical thinking in students from the very beginning years of childhood. There may be some limitation behind less utilization of pedagogical practices in real classrooms such as overcrowded classroom, overloaded curriculum, lack of proper training of teachers, lack of infrastructures, over burden teachers and administrative ignorance towards innovation. This paper discussed some of the approaches of innovative mathematics teaching which are concerned with the proposed approach of NEP 2020. With the help of these approaches the teacher will be able to shift the learning from memorization to deep understanding of concepts that make mathematical learning joyful rather than burden. The professional development programs such as pre- service and in-service teachers training may shift an ordinary teacher to an innovative teacher.

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